General Solution to Bond Rolling of Unbounded Sandwich Sheet with Outer Soft and Inner Hard Layers Considering Coulomb Friction

[1]S.C. Pan, [2]G.Y. Tzou, [3]Y. M. Hwang

[1]Department of Vehicle Engineering, Yung-Ta Institute of Technology, Taiwan

[2] Department of Mechanical and Automation Engineering, Kao Yuan University, Luzhu, Kaohsiung, Taiwan

[3]Department of Mechanical and Electro-Mechanical Engineering, Sun Yat-sen University, Taiwan

Email:scpan@seed.net.tw

Abstract - This study derives the stress field of the bond rolling of unbounded sandwich sheet with outer hard and inner soft layers at the roll gap considering Coulomb friction between the roll and the sandwich, and Coulomb friction at the interface of the unbounded region. Due to the sandwich sheet unbounded before rolling, three-layer sheets are not bonded firmly. The neutral point between the roll and the sandwich sheet, the rolling pressure distribution along contact interface between the roll and sandwich sheet, the horizontal stress of whole sandwich sheet, the horizontal stresses in the component layers of sandwich, the shear stress at the interface of sandwich sheet, the rolling force, and rolling torque, etc. are effectively calculated via this model. Furthermore, it is of great important to obtain the bonding point at the interface and the thickness ratio of sandwich sheet at the exit from this study. Additionally, the bonding conditions of the unbounded sandwich sheet are found to avoid the failure in bond rolling. This study proposed is suitable for the on-line bond rolling; it offers useful knowledge to conduct the experimental bonding conditions.

Keywords - Bond rolling; Bonding point; Thickness ratio of sandwich sheet

I. INTRODUCTION

The production of sandwich sheet by rolling processes, which are more efficient and economical approaches compared to other types of processes, has become an increasingly important subject of study. The difference between the flow stresses of sheets makes the analysis more complicated, therefore the analyses concerning the stress field and plastic deformation mechanism of sheets at the roll gap have not been thoroughly explored so far. A few studies of the manufacturing processes of sandwich sheet rolling have been presented. Most of them, however, are experimental works. Among the investigations in the sandwich sheet rolling, some authors [1-2] have developed for the prediction of rolling force and other parameters using the conventional slab analysis of sandwich sheet bonded before rolling, that combining the Runge Kutta method to solve the governing equations. Moreover, the experimental measurements of the bonded sandwich sheet have been performed. However, the conventional slab analysis of the bonded sandwich sheet is time-consuming and inconvenient, not to mention the unbounded sandwich bond rolling. An admissible velocity field based on upper bound method was proposed to examine the deformation mechanism at the roll gap by Kiuchi et al. [6]. Hwang et al. [7-8] used the stream function combining the upper bound method to establish an admissible velocity field to explore the plastic deformation behavior of sandwich sheet unbounded before rolling. Experiments on unbounded sandwich sheet rolling were also conducted by employing aluminum, mild steel and stainless as layers of sandwich sheets. Then Hwang et al. [9] developed an extended analytical and experimental study for complex rolling of unbounded sandwich sheets based on the stream function method considering constant shear friction. It is indicated that the bonding length increases with an increase in the reduction and the thickness ratio at the exit is smaller than that at the entrance due to the unbounded sandwich sheet with outer soft and inner hard layers. These results have an agreement with an analytical study [10, 11]. On the contrary, the thickness ratio at the exit for the outer hard and inner soft is larger than that at the entrance based on constant shear friction [12]. The major shortcoming of upper bound theorem or stream function method is that the stress distributions are unable to obtain, besides the CPU time required is more so as not to be suitable for on-line rolling industry about predicting rolling force and torque. For predicting rolling force and rolling torque, the slab method is a good technique if without using Runge Kutta method. Now, in this study, an analytical study for the bond rolling of unbounded sandwich sheet with outer hard and inner soft layers considering Coulomb friction is proposed to explore the various stress distributions at the roll gap. Integral analytical results can be made. In a word, this study is more suitable for the rolling industry.

II. MODELING

Fig.1 shows Schematic illustration of bond rolling of unbounded sandwich sheet. The roll radii and roll speeds for the two rolls are the same. The single neutral point (x_n) is generated at the roll gap. Due to the sandwich sheet unbounded before rolling, as the sandwich sheet is initially bit into the roll gap, the soft sheet (layer c) is yielded, however, the hard sheet (layer m) is not yet yielded. Thus, this region (zone I) belongs to the region of the unbounded sandwich sheet. The slab stress state in zone I is demonstrated in Fig. 2, where the shear stress at the interface (τ_m) is $\mu_m p_m$. As the hard sheet is yielded, the sandwich sheet is bonded completely, then the bonding point (x_b) is obtained.

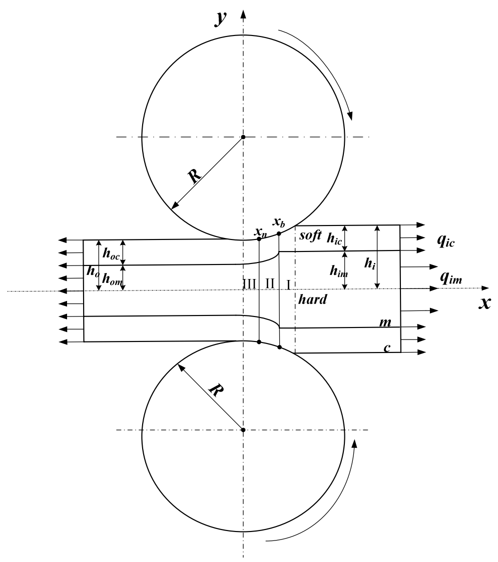

Fig. 1 Schematic diagram of bond rolling of unbounded sandwich sheet with inner hard and outer soft layers.

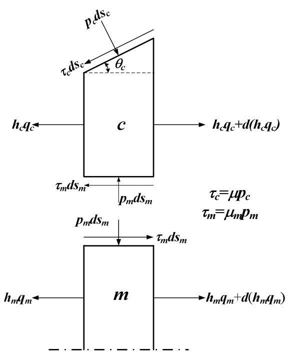

Fig. 2 Slab stress state of unbounded sandwich sheet in zone I.

The plastic deformation region at the roll gap can be divided into three distinct regions, zone I $(x_b \leq x \leq L)$ for the entrance unbounded region, zone II $(x_n \leq x \leq x_b)$ for the entrance bounded region and zone III $(0 \leq x \leq x_n)$ for the exit bounded region, where the friction forces for the zone I and II are opposite to the friction force for zone III. The slab state of sandwich sheet in zone II and III is shown in Fig. 3; where the shear stress at the interface (τ_m) is determined by the model. Zone I is the unbounded region of sandwich sheet, the soft sheet (layer c) is yielded, where the yield criterion is $p+q_c < 2k_c$, but the hard sheet (layer m) is not yielded, where $p+q_m = 2k_m$. Thus the governing equation in zone I for the soft sheet is first established.

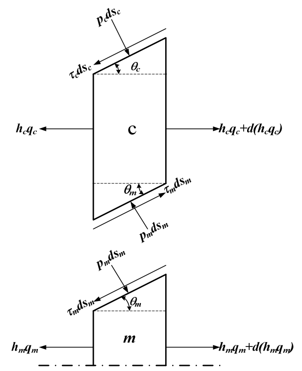

Fig. 3 Slab stress state of unbounded sandwich sheet in zone II and III.

A. Zone I – Equilibrium Equations

From Fig. 1, the equilibrium equations can be derived. Layer ***c*** is yielded: $p+q_c = 2k_c$, and then $dq_c = -dp$

The horizontal force equilibrium equation:

$$\frac{d(h_c q_c)}{dx} + p_c \tan\theta_c - \tau_c - \tau_m = 0 \quad (1)$$

The vertical force equilibrium equation:

$$p = p_m = p_c + \tau_c \tan\theta_c \quad (2)$$

Combining the horizontal and vertical force equilibrium equations, then becomes:

$$\frac{d\left(h_c q_c\right)}{dx} + p\left(\tan\theta_c - \mu_c - \mu_m\right) = 0 \quad (3)$$

where

$$\tan\theta_c = \frac{x}{R} \quad , \quad h = h_o + \frac{x^2}{2R} = h_c + h_{im} \quad , \quad \frac{dh}{dx} = \frac{dh_c}{dx} = \frac{x}{R}$$

$$h_c = \frac{2R(h_o - h_{im}) + x^2}{2R} \quad , \quad \tau_m = \mu_m p_m \quad , \quad \tau_c = \mu_c p_c$$

By using the yield criterion, geometrical conditions, we can obtain the Governing equation:

$$-h_c \frac{dp}{dx} + \frac{2k_c x}{R} - \mu_e p = 0 \quad (4)$$

where $p = 2k_C f$, $\mu_e = \left(\mu_C + \mu_m\right)$

B. Zone I - Stresses

Equation (4) is the governing equation of the bond rolling of the sandwich sheet with outer soft and inner hard layers. By Eq. (3), $2R(h_o - h_{im})$ can be as denominator; where $h_o > h_{im}$ for case I; $h_o = h_{im}$ for case II; $h_o < h_{im}$ for case III. Before solving Eq. (4) three cases need to be determined

There are three conditions of (p_I) **:**

Case I: $h_o > h_{im}$ as $\beta_i > r$

rolling stress (p_I) in zone I :

$$p_1 = 2k_c\left[\frac{2}{a_1}(\omega_1 - \frac{1}{a_1}) + c_1 e^{-a_1\omega_1}\right] \quad (5)$$

where

$$\omega_1 = \tan^{-1}\frac{x}{\sqrt{D_1}} \qquad a_1 = \mu_e \frac{2R}{\sqrt{D_1}}$$

$$D_1 = 2R(h_o - h_{im})$$

Case III: $h_o < h_{im}$ as $\beta_i < r$

rolling stress (p_1) in zone I :

$$p_1 = 2k_c\left[\frac{2}{a_1} + \frac{4\omega_1^2}{3(a_1+2)} + \frac{c_1 2^{a_1}}{\omega_1^{a_1}}\right] \quad (6)$$

where

$$\omega_1 = \cos^{-1}\frac{\sqrt{D_3}}{x},\ a_1 = \frac{2\mu_e R}{\sqrt{D_3}},\ D_3 = -2R(h_o - h_{im})$$

Layer *m* is not yielded: $p + q_m < 2k_m$

since $h_m = h_{im}$, then $\frac{dh_m}{dx} = 0$

Combined the horizontal and vertical force equilibrium equations of layer *m*:

$$\frac{d(h_m q_m)}{dx} + \tau_m = 0 \quad (7)$$

By using the yield criterion, geometrical conditions, we can obtain the horizontal stress of layer m, q_m:

Case I: $h_o > h_{im}$ as $\beta_i > r$

$$q_m = -\frac{2k_c\mu_m\sqrt{D_1}}{h_{im}}(Q_A + Q_B) + c_q \quad (8)$$

where

$$Q_A = \frac{\omega^4}{2a_1} - \frac{2\omega^3}{3a_1^2} + \frac{\omega^2}{a_1} - \frac{2\omega}{a_1^2},\ Q_B = -\frac{c_1 e^{-a_1\omega}}{a_1}(\omega^2 + \frac{2\omega}{a_1} + \frac{2}{a_1^2} + 1),$$

$$\omega = \tan^{-1}\frac{x}{\sqrt{D_1}}$$

$$D_1 = 2R(h_o - h_{im})$$

Case III: $h_o < h_{im}$ as $\beta_i < r$

$$q_m = \frac{2k_c\mu_m\sqrt{D_1}}{h_{im}}(Q_C + Q_D) + C_q \quad (9)$$

where

$$Q_C = \frac{\omega^2}{a_1} + \frac{\omega^4}{3(a_1+2)} + \frac{c_1 2^{a_1}\omega^{(2-a_1)}}{(2-a_1)}$$

$$Q_D = \frac{\omega^4}{4a_1} + \frac{\omega^6}{9(a_1+2)} + \frac{c_1 2^{(a_1-1)}\omega^{(4-a_1)}}{(4-a_1)}$$

C. Boundary Conditions

The plastic deformation region at the roll gap can be divided as three zones, so the boundary conditions for three zones have to be discussed.

(a) zone I $(x_b \le x \le L)$: $\tau_c = \mu_c p_c$

Case I: $h_o > h_{im}$ as $\beta_i > r$

(1) $x = L$, $q_c = q_{ic}$, $p_i = 2k_c - q_{ic}$

By using the boundary conditions, c_1 can be determined.

$$c_1 = e^{a_1\omega_{i1}}[1 - \frac{q_{ic}}{2k_c} - \frac{2}{a_1}(\omega_{i1} - \frac{1}{a_1})] \quad (10)$$

where

$$\omega_{i1} = \tan^{-1}\frac{L}{\sqrt{D_1}}$$

Case III: $h_o < h_{im}$ as $\beta_i < r$

$$c_1 = \frac{\omega_{i1}^{a_1}}{2^{a_1}}[-\frac{2}{a_1} - \frac{4\omega_{i1}^2}{3(a_1+2)} + 1 - \frac{q_{ic}}{2k_c}] \quad (11)$$

where

$$\omega_{i1} = \cos^{-1}\frac{\sqrt{D_3}}{L}$$

Case I: $h_o > h_{im}$ as $\beta_i > r$

(2) $x = L$, $q_m = q_{im}$

Using the boundary condition, c_q can be obtained.

$$c_q = q_{im} - \frac{2k_c\mu_m\sqrt{D_1}}{h_{im}}(Q_{iA} + Q_{iB}) \quad (12)$$

where

$$\omega_{i1} = \tan^{-1}\frac{L}{\sqrt{D_1}},\ Q_{iA} = \frac{\omega_{i1}^4}{2a_1} - \frac{2\omega_{i1}^3}{3a_1^2} + \frac{\omega_{i1}^2}{a_1} - \frac{2\omega_{i1}}{a_1^2}$$

$$Q_{iB} = -\frac{c_1 e^{-a_1\omega_{i1}}}{a_1}(\omega_{i1}^2 + \frac{2\omega_{i1}}{a_1} + \frac{2}{a_1^2} + 1)$$

Case III: $h_o < h_{im}$ as $\beta_i < r$

$$c_q = q_{im} + \frac{2k_c\mu_m\sqrt{D_1}}{h_{im}}(Q_{iC} + Q_{iD}) \quad (13)$$

where

$$\omega_{i1} = \cos^{-1}\frac{\sqrt{D_3}}{L},\ Q_{iC} = \frac{\omega_{i1}^2}{a_1} + \frac{\omega_{i1}^4}{3(a_1+2)} + \frac{c_1 2^{a_1}\omega_{i1}^{(2-a_1)}}{(2-a_1)},$$

$$Q_{iD} = \frac{\omega_{i1}^4}{4a_1} + \frac{\omega_{i1}^6}{9(a_1+2)} + \frac{c_1 2^{(a_1-1)}\omega_{i1}^{(4-a_1)}}{(4-a_1)}$$

By using $hq = h_c q_c + h_m q_m$, the horizontal stress of the whole clad sheet (q_1) in Zone I can be obtained.

$$q_1 = \frac{h_1 q_{ic} + h_2 q_{im}}{h} = \frac{h_c q_{ic} + h_{im} q_{im}}{h_c + h_{im}} \quad (14)$$

(3) Bonding Point (x_b)

At $x = x_b$, the hard layer is yielded and the yielding condition is expressed as $p + q_m = 2k_m$.

Case I: $h_o > h_{im}$ as $\beta_i > r$

$$[\frac{2}{a_1}(\omega_{b1} - \frac{1}{a_1}) + c_1 e^{-a_1\omega}] - \frac{\mu_m\sqrt{D_1}}{h_{im}}(Q_{bA} + Q_{bB}) + c_q = \frac{k_m}{k_c} \quad (15)$$

where

$$\omega_{b1} = \tan^{-1}\frac{x_b}{\sqrt{D_1}} \ , \ Q_{bA} = \frac{\omega_{b1}^{\ 4}}{2a_1} - \frac{2\omega_{b1}^{\ 3}}{3a_1^{\ 2}} + \frac{\omega_{b1}^{\ 2}}{a_1} - \frac{2\omega_{b1}}{a_1^{\ 2}}$$

$$Q_{bB} = -\frac{c_1 e^{-a_1\omega_{b1}}}{a_1}(\omega_{b1}^{\ 2} + \frac{2\omega_{b1}}{a_1} + \frac{2}{a_1^{\ 2}} + 1)$$

Case III: $h_o < h_{im}$ as $\beta_i < r$

$$\left[\frac{2}{a_1} + \frac{4\omega_{b1}^{\ 2}}{3(a_1+2)} + \frac{c_1 2^{a_1}}{\omega_{b1}^{\ a_1}}\right] + \frac{\mu_m\sqrt{D_3}}{h_{im}}(Q_{bC} + Q_{bD}) + c_q = \frac{k_m}{k_c} \tag{16}$$

where

$$\omega_{b1} = \cos^{-1}\frac{\sqrt{D_1}}{x_b} \ , \ Q_{bC} = \frac{\omega_{b1}^{\ 2}}{a_1} + \frac{\omega_{b1}^{\ 4}}{3(a_1+2)} + \frac{c_1 2^{a1}\omega_{b1}^{(2-a_1)}}{(2-a_1)}$$

$$Q_{bD} = \frac{\omega_{b1}^{\ 4}}{4a_1} + \frac{\omega_{b1}^{\ 6}}{9(a_1+2)} + \frac{c_1 2^{(a1-1)}\omega_{b1}^{(4-a_1)}}{(4-a_1)}$$

The specific shear stress $(\tau_m / k_c)_I$ in zone I is:

$$(\tau_m / k_c)_I = \frac{\mu_m p}{k_c} \tag{17}$$

(b) zone II $(x_n \le x \le x_b)$

In zone II and zone III, three layers are bonded. however, thickness ratios at entrance and exit $(\beta_i \ \& \ \beta_o)$ & are different.

$\beta_i = \frac{h_{ic}}{h_i}$ and $\beta_o = \frac{h_{bc}}{h_b} = \frac{h_{oc}}{h_o}$ cannot be same, that is β_o is less than β_i .

The rolling pressure distribution (p_{II}) in Zone II is:

$$p_{II} = 2k_c\left[\frac{2\alpha}{a_2}(\omega - \frac{1}{a_2}) + c_2 e^{-a_2\omega}\right] \tag{18}$$

where

$$\alpha = \beta_o - (1-\beta_o)\frac{k_m}{k_c} \ , \ \omega = \tan^{-1}\frac{x}{\sqrt{D}} \ , \ a_2 = \mu_c\sqrt{\frac{2R}{h_o}}$$

$$D = 2Rh_o$$

Case I: $h_o > h_{im}$ as $\beta_i > r$

$$c_2 = e^{a_2\omega_b}\left[\frac{2}{a_1}\left(\omega_{b1} - \frac{1}{a_1}\right) + c_1 e^{-a_1\omega_{b1}} - \frac{2\alpha}{a_2}\left(\omega_b - \frac{1}{a_2}\right)\right] \tag{19}$$

where $\omega_b = \tan^{-1}\frac{x_b}{\sqrt{D_1}}$

Case III: $h_o < h_{im}$ as $\beta_i < r$

$$c_2 = e^{a_2\omega_b}\left[\frac{2}{a1} + \frac{4\omega_{b1}^2}{3(a_1+2)} + \frac{c_1 2^{a1}}{\omega_{b1}^{a1}} - \frac{2\alpha}{a_2^2}(a_2\omega_b - 1)\right] \tag{20}$$

The specific shear stress $(\tau_m)_{II}$ in Zone II is:

$$(\tau_m / k_c)_I = \frac{\mu_m p}{k_c}$$

$$(\tau_m)_{II} = \frac{\frac{2k_m(1-\beta_o)x}{R} - (1-\beta_o)h(\frac{dp}{dx})_{II}}{\Theta_m} \tag{21}$$

where

$$\frac{dp}{dx} = 2k_c\left[\frac{\sqrt{D}}{D+x}(-a_2 c_2^{-a_2\omega_i} + \frac{2\alpha}{a_2})\right]$$

$$\Theta_m = \tan^2\theta_m + 1 = \frac{(1-\beta_o)^2 x^2}{R^2} + 1$$

At $x = x_b$, $p_I = p_{II}$

Case I: $h_o > h_{im}$ as $\beta_i > r$

$$\left[\frac{2}{a_1}(\omega_{b1} - \frac{1}{a_1}) + c_1 e^{-a_1\omega_{b1}}\right] = \left[\frac{2\alpha}{a_2}(\omega_b - \frac{1}{a_2}) + c_2 e^{-a_2\omega_b}\right] \tag{22}$$

where

$$\omega_{b1} = \tan^{-1}\frac{x_b}{\sqrt{D_1}}$$

Case III: $h_o < h_{im}$ as $\beta_i < r$

$$\left[\frac{2}{a_1} + \frac{4\omega_{b1}^{\ 2}}{3(a_1+2)} + \frac{c_1 2^{a_1}}{\omega_{b1}^{\ a_1}}\right] = \left[\frac{2\alpha}{a_2}(\omega_b - \frac{1}{a_2}) + ce^{-a_2\omega_b}\right] \tag{23}$$

where

$$\omega_{b1} = \tan^{-1}\frac{\sqrt{D_3}}{x_b}$$

(c) zone III $(0 \le x \le x_b)$

The rolling pressure distribution in zone III is expressed as:

$$p_{III} = 2k_c\left[\frac{2\alpha}{a_3}(\omega - \frac{1}{a_3}) + c_3 e^{-a_3\omega}\right] \tag{24}$$

where

$$\alpha = \beta_o - (1-\beta_o)\frac{k_m}{k_c} \quad \omega = \tan^{-1}\frac{x}{\sqrt{D}} \quad a_3 = \mu_c\sqrt{\frac{2R}{h_o}}$$

At $x = 0$, $q = q_o$, $p_o = 2k_e - q_o$, so

$$c_3 = \alpha - \frac{q_0}{k_c} + \frac{2\alpha}{a_3^2} \tag{25}$$

The specific shear stress $(\tau_m)_{III}$ in Zone III is:

$$(\tau_m)_{III} = \frac{\frac{2k_m(1-\beta_o)x}{R} - (1-\beta_o)h\left(\frac{dp}{dx}\right)_{III}}{\Theta_m} \tag{26}$$

where

$$\left(\frac{dp}{dx}\right)_{III} = 2k_c\left[\frac{\sqrt{D}}{D+x}\left(-a_3 c_3^{-a_3\omega_n} + \frac{2\alpha}{a_3}\right)\right]$$

$$\Theta_m = \tan^2\theta_m + 1 = \frac{(1-\beta_o)^2 x^2}{R^2} + 1$$

D. Neutral Point (x_n)

At $x = x_n$, $p_{III} = p_{II}$

$$\left[\frac{2\alpha}{a_3}(\omega_n - \frac{1}{a_3}) + c_3 e^{-a_3\omega_n}\right] = \left[\frac{2\alpha}{a_2}(\omega_n - \frac{1}{a_2}) + c_2 e^{-a_2\omega_n}\right] \tag{27}$$

where

$$c_3 = \alpha - \frac{q_o}{k_c} + \frac{2\alpha}{a_3^2} \qquad \omega_n = \tan^{-1}\frac{x_n}{\sqrt{D_1}}$$

$(\boldsymbol{x_n})$ can be easily solved using the bisection method.

E. Rolling Force

The rolling force can be found by integrating the normal rolling pressure over the arc length of contact. Thus, the rolling force per unit width, P , is given by:

$$P = \int_0^{x_n} p_{III}\,dx + \int_{x_n}^{x_b} p_{II}\,dx + \int_{x_b}^{L} p_I\,dx = p_{III} + p_{II} + p_I \tag{28}$$

where

$$p_{III} = III_1 + III_2$$

$$III_1 = -\frac{c_3}{a_3} e^{-a_3 \omega_n} \left[\frac{2}{a_3^2} + \frac{2\omega_n}{a_3} + \omega_n^2 \right] + \frac{1}{a_3} \left[-\frac{2\alpha\omega_n}{a_3} + \alpha\omega_n^2 - \frac{2\alpha\omega_n^3}{3a_3} + \frac{\alpha\omega_n^4}{2} + 1 \right]$$

$$III_2 = \frac{c_3}{a_3} \left(\frac{2}{a_3^2} + 1 \right)$$

$$P_{II} = II_1 + II_2$$

$$II_1 = -\frac{c_2}{a_2} e^{-a_2 \omega_b} \left[\frac{2}{a_2^2} + \frac{2\omega_b}{a_2} + \omega_b^2 + 1 \right] + \frac{1}{a_2} \left[-\frac{2\alpha\omega_b}{a_3} + \alpha\omega_b^2 - \frac{2\alpha\omega_b^3}{3a_2} + \frac{\alpha\omega_b^4}{2} \right]$$

$$II_2 = \frac{c_2}{a_2} e^{-a_2 \omega_n} \left[\frac{2}{a_2^2} + \frac{2\omega_n}{a_2} + \omega_n^2 + 1 \right] + \frac{1}{a_2} \left[-\frac{2\alpha\omega_n}{a_3} + \alpha\omega_n^2 - \frac{2\alpha\omega_n^3}{3a_2} + \frac{\alpha\omega_n^4}{2} \right]$$

c_1 is shown as two cases:

Case I : $h_o > h_{im}$ as $\beta_i > r$

$$P_I = I_1 + I_2$$

$$I_1 = \frac{c_1}{a_1} e^{-a_2 \omega_{i1}} \left[\frac{2}{a_1^2} + \frac{2\omega_{i1}}{a_1} + \omega_{i1}^2 + 1 \right] + \frac{1}{a_1} \left[-\frac{2\omega_{i1}}{a_1} + \omega_{i1}^2 - \frac{2\omega_{i1}^3}{3a_1} + \frac{\omega_{i1}^4}{2} \right]$$

$$I_2 = -\frac{c_1}{a_1} e^{-a_2 \omega_{b1}} \left[\frac{2}{a_1^2} + \frac{2\omega_{b1}}{a_1} + \omega_{b1}^2 + 1 \right] + \frac{1}{a_1} \left[-\frac{2\omega_{b1}}{a_1} + \omega_{b1}^2 - \frac{2\omega_{b1}^3}{3a_1} + \frac{\omega_{b1}^4}{2} \right]$$

Case III : $h_o < h_{im}$ as $\beta_i < r$

$$P_I = I_1 + I_2$$

$$I_1 = \frac{\omega_{i1}^6}{9(a_1+2)} + \frac{\omega_{i1}^4}{3(a_1+2)} + \frac{\omega_{i1}^4}{4a_1} + \frac{\omega_{i1}^2}{a_1} + \frac{c_1 2^{a_1} \omega_{i1}^{4-a_1}}{4-a_1} + \frac{c_1 2^{a_1} \omega_{i1}^{2-a_1}}{2-a_1}$$

$$I_2 = \frac{\omega_{b1}^6}{9(a_1+2)} + \frac{\omega_{b1}^4}{3(a_1+2)} + \frac{\omega_{b1}^4}{4a_1} + \frac{\omega_{b1}^2}{a_1} + \frac{c_1 2^{a_1} \omega_{b1}^{4-a_1}}{4-a_1} + \frac{c_1 2^{a_1} \omega_{b1}^{2-a_1}}{2-a_1}$$

F. Rolling Torque

The rolling torque (T), can be calculated by integrating the moment of the roll axis. Therefore:

$$T = 2R\left(-\mu_c P_{III} + \mu_c P_{II} + \mu_c P_I\right) = 2R\mu_c\left(P_{II} + P_I - P_{III}\right)$$

G. Case II Model

From Fig. 1, Layer ***c*** is yielded :

$p + q_c = 2k_c$, then $dq_c = -dp$

Horizontal force equilibrium equation:

$$\frac{d(h_c q_c)}{dx} + p_c \tan\theta_c - \tau_c - \tau_m = 0 \tag{29}$$

Vertical force equilibrium equation:

$$p = p_m = p_c + \tau_c \tan\theta_c \tag{30}$$

Combining the horizontal and vertical force equilibrium equations; then becomes:

$$\frac{d(h_c q_c)}{dx} + p(\tan\theta_c - \mu_c - \mu_m) = 0 \tag{31}$$

By using the yield criterion, geometrical conditions, we can obtain the governing equation:

$$\frac{dp}{dx} = -\frac{2R}{D_1 + x^2}\mu_e p + \frac{4k_c}{D_1 + x^2}x \tag{32}$$

where

$p = 2k_c f$, $\mu_e = (\mu_c + \mu_m)$

Case II : $h_o = h_{im}$ as $\beta_i = r$, $D_1 = 0$

(1) At $x = L$, $q_c = q_{ic}$, $p_i = 2k_c - q_{ic}$

The Eq. (32) can not be derived directly by the analytical approach, so the numerical solution can be obtained by using the Software "Mathematica".

Layer *m* is not yielded :

$p + q_m < 2k_m$, $h_o = h_{im}$, $\frac{dh_m}{dx} = 0$

From equations of equilibrium

$$h_m \frac{dq_m}{dx} = -\mu_m p \tag{33}$$

At $x = L$, $q_m = q_{im}$

The numerical solution of q_m can be determined.

(2)At $x = x_b$, $p + q_m = 2k_m$

The numerical solutions of x_b , $q_c/2k_c$, $q_m/2k_m$, $p/2k_c$, $q/2k_c$, and τ_m/k_c can be calculated by using the Software "Mathematica".

H. Variation ratio of clad thickness deformation

In order to effectively evaluate whether the clad sheet has deformed uniformly, it is necessary to define the variation ratio of clad thickness deformation (λ):

$$\lambda = \left| \frac{\beta_o - \beta_i}{\beta_i} \right| \times 100\% \tag{34}$$

When λ is large, it represents the non-uniform deformation, it indicates the clad thickness ratio at the exit is higher than that at the entrance.

III. RESULT AND DISCUSSIONS

Fig. 4 shows various stress distributions for various initial thickness ratios $(\beta_i = h_{ic}/h_i)$. As the initial thickness ratios at the entrance increases, it indicated more hard layer fraction is occurred, all stresses except for τ_m/k_c decrease, and the bonding point is generated early.

Fig. 5 shows effects of initial thickness ratio upon the rolling force and rolling torque for various reductions. From this figure, the rolling force and rolling torque increase with increasing the reductions (r) under the same thickness ratio (β_i) at the entrance. The rolling force and rolling torque decrease with increasing thickness ratio (β_i) at the entrance under same reductions (r).

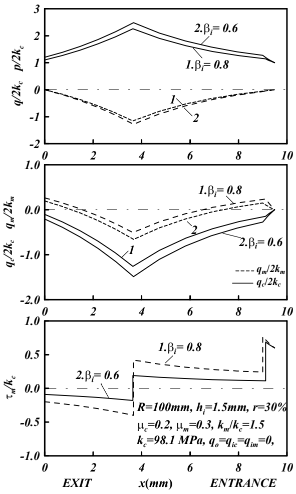

Fig. 4 various stress distributions for various initial

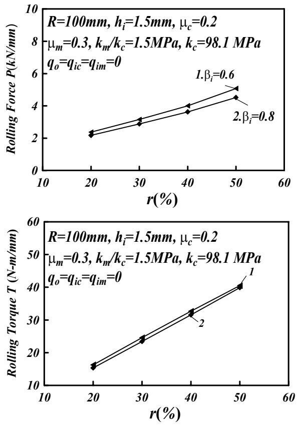

Fig. 5 effects of initial thickness ratio upon the rolling force and rolling torque for various

Fig. 6 shows effects of initial thickness ratio on λ and x_b / L for various shear yield stress ratios. Under the fixed initial thickness ratio, shear yield stress ratio (k_m/k_c) increases, λ also increases, but x_b / L decreases. As x_b / L increases, it indicates the sandwich sheet would be bonded early. From this figure, it reveals that the inverse point is changing with increasing k_m/k_c . As β_i increases, λ and x_b / L decrease until the inverse point is occurred. After the inverse point, the situation is opposite to the above.

Fig. 7 shows effects of effects of initial thickness ratio on λ and x_b / L for various reductions. x_b is for the bonding point or bonding length, and x_b / L is for a relative bonding length. As the bonding length (x_b / L) becomes large, it states that sandwich clad sheet is easier to bonding. Under the fixed initial thickness ratios (β_i) , λ and x_b / L decrease with increasing reductions (r) . With increasing x_b / L it states sandwich sheet is bonded early, and it also states sandwich sheet deforms seriously with increasing λ . The effect of x_b / L for the initial thickness ratios (β_i) would be observed from this figure. As $\beta_i = 0.60$, increasing β_i would decrease x_b / L , and vice versa. It states that the inverse point is $\beta_i = 0.60$.

IV. CONCLUSIONS

The main analytical results are as follows:

(1) As the reduction (r), the interface frictional coefficient (μ_m) increase, and the shear stress ratio decreases, the sandwich sheet would be bonded early between rolls. The λ and x_b / L will be affected by the initial thickness ratio.

(2) As the frictional coefficient at the interface (μ_m) increases, the shear yield stress ratio decreases, the thickness ratio at the exit (β_o) is closed to that at the entrance (i.e. λ is smaller). Namely the deformation between the three-layers is more uniform.

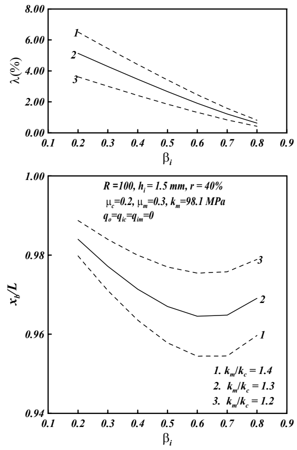

Fig. 6 effects of initial thickness ratio on λ and x_b / L for various shear yield stress ratio.

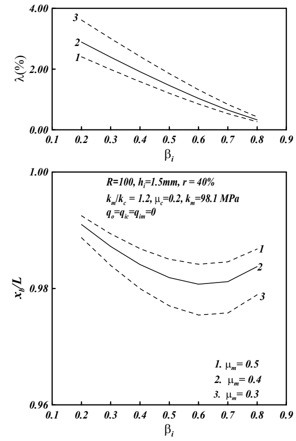

Fig. 7 effects of initial thickness ratio on λ and x_b / L for various reductions.

REFERENCES

[1] R. R. Arnold, P. W. Whitton, "Stress and Deformation Studies for Sandwich Rolling Hard Metals," *Proc. Instn. Mech. Engrs*., 173 (1959) 241-256.

[2] A. A. Afonja, D. H.. Sansome "A Theoretical Analysis of The Sandwich Rolling Process," I*nt. J. Mech. Sciences*, 15 (1973) 1-14.

[3] E. Tanaka, T. Kukuda, Y. Suzuki, M. Nakamoto, "Cold Clad Rolling of Copper-Steel-Copper," *J. Jap. Soc. Technol. Plasticity,* 12 (1971) 622-630.

[4] T. Shirota, T. Dendo, "Effect of Rolling Conditions on Bonding Behavior in Cold Clad Rolling by Planetary mill," *J. Jap. Soc. Technol. Plasticity,* 24 (1983) 53-58.

[5] M. Denchiyuu, "Sandwich Rolling Analysis and Experiments," Proceeding of The S-54 Spring Conference, *J. Jap. Soc. Technol. Plasticity,* (1984) 557-560.

[6] M. Kiuchi, Y. M. Hwang, K. Shintani, "Analysis and Experiment on Sandwich Sheet Rolling," *J. Jap. Soc. Technol. Plasticity,* 31 (1990) 1445-1450.

[7] Y. M. Hwang, H. H. Hsu, H. J. Lee, "Analysis of Sandwich Sheet Rolling by Stream Function Method," *Int. J. Mech. Sciences,* 37 (1995) 297-315.

[8] Y. M. Hwang, H. H. Hsu, H. J. Lee, "Analysis of Plastic Instability During Sandwich Sheet Rolling," *Int. J. Mach. Tools. Manufacture,* 36 (1996) 47-62.

[9] Y. M. Hwang, H. H. Hsu, Y. L. Hwang, "Analytical and Experimental Study on Bonding Behavior at The Roll Gap During Complex Rolling of Sandwich Sheets," *Int. J. Mech. Sciences,* 42 (2000) 2417-2437.

[10] G. Y. Tzou, M. N. Huang, "Study on The Cold-and-Hot Bond Rolling of Unbounded Sandwich Sheet With Outer Soft and Inner Hard Layers," Proceeding of The 4th International Conference on ESAFORM2001, II (2001) 859-862.

[11] M. N. Huang, G. Y. Tzou, H. Y. Hsu, C. F. Liu, K. C. Liang, "General Solutions of The Bond Rolling of Unbounded Sandwich Sheet with Outer Soft and Inner Hard Layers," *Journal of Chinese Society Mechanical Engineering,* 26 (2005) no.5 629-635.

[12] G. Y. Tzou, M. N. Huang, "General Solutions of the Bond Rolling of Unbounded Sandwich Sheet with Outer Hard and Inner Soft Layers," *Journal of Chinese Society Mechanical Engineering,* .27 (2006) No.1 93-98.