Performance Analysis of Magnetic Hydrodynamic Tilted Bearing with Surface Roughness

Hsiu-Lu Chiang, Tsu-Liang Chou, Ching-Been Yang, Teng-Lang Feng*

Department of Mechanical Engineering, Nanya Institute of Technology

**rotex@nanya.edu.tw*

Abstract- The performances characteristics of magnetic hydrodynamic tilted bearing with surface roughness lubricated with ferrofluid are studied in this study. To explore the effects of ferrofluid to the bearings, the Shah's theoretical model and the modified items of characteristics related to magnetic ferrofluid are adopted. As the affections of surface roughness to the bearings, the stochastic Christensen Reynolds' equation is applied; meanwhile, with the consideration of the non-zero mean α, variance σ and skewness ε. According to the results, comparing with the Newtonian fluids, the tilted bearing lubricated with magnetic ferrofluid has the higher built-up pressure distribution and load-carrying capacities. As the non-zero mean α of the surface roughness increases, the responding time decreases. On the other hand, increases the variation σ, decreases the responding time in longitudinal surface roughness; whereas, the transverse type has the inverse trend of responding time when variation increases.

***Keywords**: ferrofluid, surface roughness, non-zero mean, tilted bearing.*

I. INTRODUCTION

The researches of slider step bearing started by Naduvinamani [1] since 1997, while the squeezing step bearing was studied first by Majumdar [2]。The traditional lubricants were used by Majumdar when characteristics of the step bearing were investigated. Shah and Bhat [3] utilized the MHD momentum equation and Maxwell's equation to analyze the performances of porous slider bearing. The researches of Majumdar were continued by Shah [4], and the magnetic ferrofluid was used as the lubricant, and a modified term was added to the expression of magnetic characteristics, thus, the properties of magnetic ferrofluid can be appeared more realistic. In the studies of Dobrica and Fillon [5], the fluid viscosity was a function of temperature, and the affections of performance to step bearing were investigated.

All abovementioned researches, the surface of bearing was assumed smooth completely. Whereas, due to the process of forming and finishing, the surfaces should be considered as rough one. Meanwhile, as the roughness is about the same order of magnitude of film thickness, the effects of surface roughness to the bearing characteristics cannot be ignored and be worth of exploring. To investigate the effects of surface roughness, based on the concepts of stochastic, the built-up hydrodynamic film pressure is a random, since the film thickness is stochastic. In accordance with the longitudinal and transverse type of surface roughness, an averaged film pressure stochastic Reynolds equation was constructed by Christensen and Tonder [6-7], and the performances of rotational bearing can be obtained. Christensen's roughness model was applied to explore the infinitive long porous journal bearing by Gururajan and Prakash [8], while Chiang *et. al.* [9] applied this roughness model to investigate the linear stability of finite journal bearing. All of the both have the fine results.

The optimum film profile for a longitudinally rough slider bearing was investigated by Andharia *et. al.* [10]. The roughness of the bearing surface is modeled by stochastic random variable with non-zero mean, variance and skewness. It is shows that, the parameters α, σ and ε characterizing the surface roughness of the bearing affect the bearing performance characteristics significantly. The effect of longitudinal surface roughness on the behavior of slider bearing with squeeze film formed by a magnetic fluid was analyzed by Deheri *et. al.* [11]. It is found that the bearing performance is significantly affected by all the three stochastic random parameters characterizing the surface roughness. The effect of surface roughness on the hydrodynamic lubrication of porous step-slider bearings was theoretical explored by Naduvinamani and Siddangouda [12]. In accordance with the results, the negatively skewed surface roughness type increases the load carrying capacity and decreases the coefficient of friction, while the adverse effects were found for the positively skewed surface roughness type.

In this study, considering the two types, longitudinal and transverse, of surface roughness, the performance characteristics of tilted bearing lubricated with magnetic ferrofluids was investigated.

II. ANALYSIS

The physical configuration of squeeze tilted bearing is plotted in Fig. 1. The film thickness can be expressed as:

$$h_s = h_x(x) + h_m(t)\ ,\quad 0 \le x \le L \tag{1}$$

The upper plane has a uniform velocity $V = dh/dt$, vertical downwards, and the length of bearing along *z* axis assumes larger than *L*.

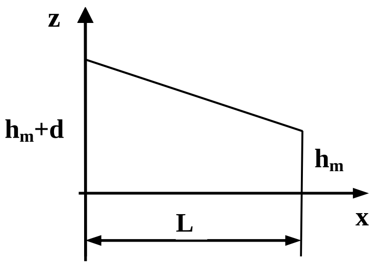

Fig. 1 Physical configuration of squeeze tilted bearing

The applied external magnetic strength *H* is in *y* direction, that is, direction having the maximal effects to the pressure distribution and bearing performance, with zero value at both end of slider bearing. Magnetic strength *H* can be defined as:

$$H^2 = Kx(L-x) \quad (2)$$

where K is a quantity satisfied homogeneous dimension of (2).

Based on the study of Shah [4], the squeezing flow equation of tilted plane is:

$$\frac{d}{dx}\left\{h^3 \frac{d}{dx}\left[p - \frac{1}{2}(1+\phi)\mu_o \chi H^2\right]\right\} = 12\zeta \frac{dh}{dt} \quad (3)$$

where ζ is the fluid viscosity, χ susceptibility and $(1+\phi)\mu_0$ permeability.

According to the Christensen stochastic roughness model, the surface roughness can be divided into two types, the longitudinal one dimensional and transverse one dimensional. On the other hand, the squeeze film thickness h can be treated as a random variable, which is composed of two parts:

$$h = h_s + \delta(x,z,\xi) \quad (4)$$

where h is the film thickness of smooth surface, δ the surface roughness measured from the smooth part and ξ a random variable.

In this study, the non-zero mean *α*, variance *σ* and skewness *ε* are all random variables, and can be defined as following, respectively.

$$\overline{\alpha} = E(\delta) \quad (5)$$

$$\overline{\sigma}^2 = E[(\delta - \overline{\alpha})^2] \quad (6)$$

$$\overline{\varepsilon} = E[(\delta - \overline{\alpha})^3] \quad (7)$$

A. *Stochastic Generalized Reynolds Equation*

On the basis of Christensen stochastic roughness model, and takes the expected values of flow equation (3), one can obtain the longitudinal one dimensional roughness flow equation of squeezing tilted bearing as:

$$\frac{d}{dx}\left\{E(h^3)\frac{d}{dx}\left[E(p) - \frac{1}{2}(1+\overline{\alpha})\mu_o \chi H^2\right]\right\} = 12\zeta \frac{dE(h)}{dt} \quad (8)$$

where $E(\cdot)$ is the expectation operator, defined as:

$$E(\cdot) = \int_{-\infty}^{\infty} (\cdot) f(\delta) d\delta \quad (9)$$

Define the following dimensionless parameters:

$$X = \frac{x}{L}, \quad V = -\frac{dh_m}{dt} = -\frac{dh_S}{dt}, \quad P^* = \frac{h_{m0}^3 E(p)}{\zeta V L^2}, \quad D = \frac{d}{h_{m0}}$$

$$\mu^* = \frac{\mu_0 K h_{m0}^3}{\zeta V}, \quad h_m^* = \frac{h_m}{h_{m0}}, \quad \alpha = \frac{\overline{\alpha}}{h_{m0}}, \quad \sigma = \frac{\overline{\sigma}}{h_{m0}}, \quad \varepsilon = \frac{\overline{\varepsilon}}{h_{m0}^3} \quad (10)$$

$$M = \frac{1}{2}(1+\phi)\chi\mu^*, \quad h^* = \frac{h_s}{h_{m0}} = (1-X)D + h_m^*$$

and (8) can be non-dimensionalized as following:

$$\frac{d}{dX}\left\{E_1 \frac{d}{dX}\left[P^* - MX(1-X)\right]\right\} = -12 \quad (11)$$

where $E_1 = h_s^{*3} + 3h_s^*(\sigma^2+\alpha^2) + 3h_s^{*2}\alpha + \varepsilon + 3\alpha\sigma^2 + \alpha^3$

On the basis of Christensen stochastic roughness model, and takes the expected values of flow equation (3), one can obtain the transverse one dimensional roughness flow equation of squeezing tilted bearing as:

$$\frac{d}{dx}\left\{\frac{1}{E(h^{-3})}\frac{d}{dx}\left[E(p) - \frac{1}{2}(1+\alpha)\mu_o \chi H^2\right]\right\} = 12\zeta \frac{d}{dt}\frac{E(h^{-2})}{E(h^{-3})} \quad (12)$$

Utilized the dimensionless parameters (10), transverse one dimensional generalized Reynolds equation (12) can be simplified as following:

$$\frac{d}{dX}\left\{\frac{1}{K_2}\frac{d}{dX}\left[P^* - MX(1-X)\right]\right\} = -12 f(h^*,\alpha,\sigma,\varepsilon) \quad (13)$$

where

$$K_2 = \frac{1}{h_s^{*3}}\left[1 - 3\frac{\alpha}{h_s^*} + 6\frac{(\sigma^2+\alpha^2)}{h_s^{*2}} - 10\frac{(\varepsilon + 3\alpha\sigma^2 + \alpha^3)}{h_s^{*3}}\right]$$

$$f(h^*,\alpha,\sigma,\varepsilon) = \frac{d}{dh^*}\left(\frac{h^{*4} - 2h^{*3}\alpha + 3h^{*2}(\sigma^2+\alpha^2) - 4h^*(\varepsilon+3\alpha\sigma^2+\alpha^3)}{h^{*3} - 3h^{*2}\alpha + 6h^*(\sigma^2+\alpha^2) - 10(\varepsilon+3\alpha\sigma^2+\alpha^3)}\right)$$

B. *Film Pressure and Load Carrying Capacity*

1) Longitudinal One Dimensional

For the longitudinal one dimensional surface roughness, at first, one can integrate (13) twice, and implies the boundary condition, *P**=0, at *X*=0,1, then, one can obtain:

$$P^* = MX(1-X) - 12\int_0^X \frac{X}{E_1}dX + K_1 \cdot \int_0^X \frac{1}{E_1}dX \quad (14)$$

$$K_1 = 12\int_0^1 \frac{X}{E_1}dX \Big/ \int_0^1 \frac{1}{E_1}dX \quad (15)$$

Once the film pressure distribution of tilted bearing obtains, the dimensionless load carry capacity of longitudinal one dimensional can be obtained by means of integration as:

$$W^* = \frac{M}{6} - 12\int_0^1\int_0^X \frac{X}{E_1}dXdX + K_1 \cdot \int_0^1\int_0^X \frac{1}{E_1}dXdX \quad (16)$$

2) Transverse One Dimensional

For the transverse one dimensional surface roughness, at first, one can integrate (13) twice, and implies the boundary condition, *P**=0, at *X*=0,1, then, one can obtain:

$$P^* = MX(1-X) - 12\int_0^X\left(K_2 \cdot \int_0^X f(h^*,\alpha,\sigma,\varepsilon)dX\right)dX + K_3\int_0^X K_2 dX \quad (17)$$

$$K_3 = 12\int_0^1\left(K_2 \cdot \int_0^1 f(h^*,\alpha,\sigma,\varepsilon)dX\right)dX \Big/ \int_0^1 K_2 dX \quad (18)$$

Once the film pressure distribution of tilted bearing obtains, the dimensionless load carry capacity of transverse one dimensional can be obtained by means of integration as:

$$W^* = \frac{M}{6} - 12\int_0^1\int_0^X\left(K_2 \cdot \int_0^X f(h^*,\alpha,\sigma,\varepsilon)dX\right)dXdX + \int_0^1\left(K_3\int_0^X K_2 dX\right)dX \quad (19)$$

3) Centre of Load Carry

As the film thickness of left end is greater than that of right end, therefore, the center of load capacity of tilted plane bearing should locate always at the right part of bearing geometry center. Definition of center of load capacity L_c is:

$$L_c = \int_0^1 X \cdot P^* dX \Big/ \int_0^1 P^* dX \tag{20}$$

The film pressure distribution is affected by the magnetic parameter M and surface roughness, thus, the position of center of load capacity L_c will change therefore. The position of L_c affected by bearing parameters will be discussed in this study.

4) Responding Time

When the load applied is steady, the time needed where bearing height approached to a specific value is called the responding time. The larger the responding time is, the better the dynamic characteristics of bearing is. To investigate the time-height relationship of the tilted slider bearing, a dimensionless responding time T is defined first as:

$$T = \frac{W^* h_{m0}^2}{\zeta\ L^3} t \tag{21}$$

From (16) and (19), the time-height relationship for the surface roughness longitudinal one dimensional and transverse one dimensional types can be achieved further.

$$\frac{dh_m^*}{dT} = -\frac{1}{W^*} \tag{22}$$

Because of the time-height relationship is a nonlinear differential equation, to get the solution, the fourth-order Runge-Kutta method accompany with the initial condition is used to solve numerically. Initial condition of the tilted slider bearing system is:

$$h_m^* = 1 \quad at \quad T = 0 \tag{23}$$

As the two types of surface roughness model are adopted in this study, consequently, one should, resting on the roughness model, substitute the dimensionless load carry capacity (16) or (19) into (22) to solve the time-height relationship of bearing system.

$$T = -\int_1^{h_m^*} W^* dh_m^* \tag{24}$$

III. RESULTS AND DISCUSSION

For a bearing fabricated with machine forming or finishing, the surface cannot perfectly smooth, there is somewhat disorder or irregular. As a result, to theoretically analyze the effects of surface roughness to the bearing performances, the stochastic random variables, non-zero mean α, variance σ and skewness ε should be taken into consideration. If the stochastic random variables, α, σ and ε are all set to zeros, one can obtain the pressure distribution from (15) and (18) of perfectly smooth bearing. The relation between built-up pressure P* and non-zero mean α, is plotted in Fig. 2. The dot line represents the pressure distribution of smooth one. Comparing with that of the smooth one, the built-up pressure of the bearing with longitudinal one dimensional roughness will decrease, and the position of maximal pressure shifts towards the outlet zone. On the contrary, the transverse one will increase the pressure distribution, while the position of maximal pressure shifts towards the inlet zone. Obviously, as one can see from Fig. 2, the larger the value of non-zero mean α is, the smaller the film pressure *P** is.

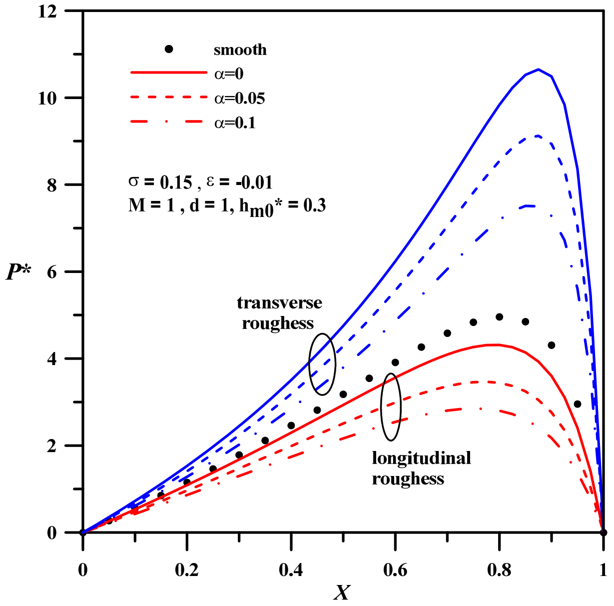

Fig. 2 Film pressure *P** as function of *X* for varied α

The variation of film pressure *P** with the effects of magnetic parameter *M* is outlined in Fig. 3. Comparing with that of the smooth one with no magnetic effect, the film pressure *P** of longitudinal one dimensional roughness will decrease, while the transverse roughness can enhance the film pressure *P**. When the tilted bearing system subjected to an external magnetic strength *M*, the built-up film pressure *P** can be improved, the higher the magnetic strength *M*, the more enhancement the film pressure has. Meanwhile, if the external magnetic strength increases, the position occurrence of the maximal film pressure P_{max}^* will shift to inlet zone.

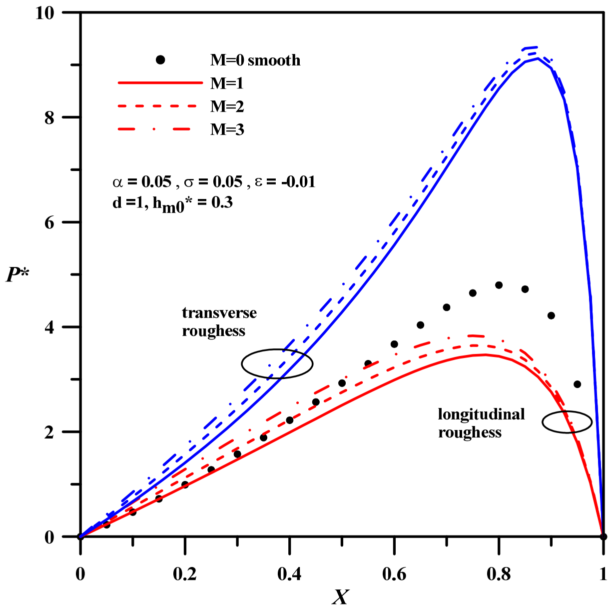

Fig. 3 Effects of magnetism *M* to film pressure *P**

The affections of stochastic non-zero mean α to the load carry capacity *W** subjected to an external magnetic strength M is graphed in Fig. 4. As abovementioned, the built-up pressure of longitudinal one dimensional roughness is lower than that of transverse one, thus, the integrated load capacity W^* of longitudinal one is lesser that of transverse one. Further, as known in Fig. 3, increasing the magnetic strength M will improve the built-up film pressure P^*, hence, the higher external magnetic strength M will develop a better load carry capacity of the tilted bearing system. Similarly, the stochastic non-zero mean α has the effect to depress the bearing pressure, so, if the surface roughness of bearing has a larger stochastic non-zero mean, the system load capacity will reduce.

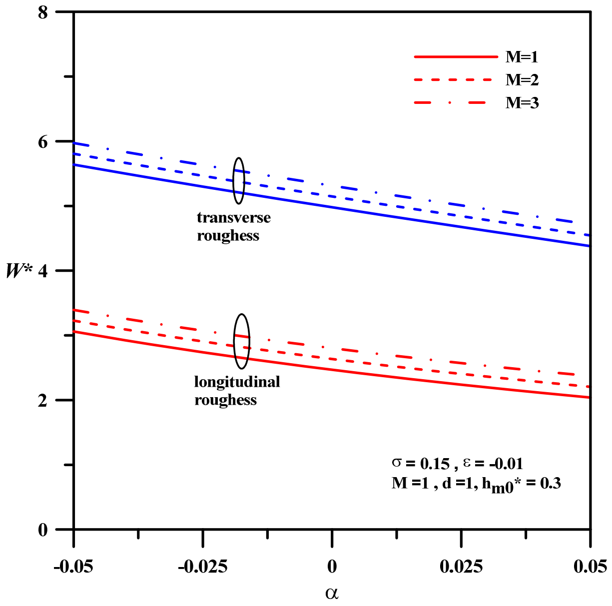

Fig. 4 Relation between W^* and α with different M

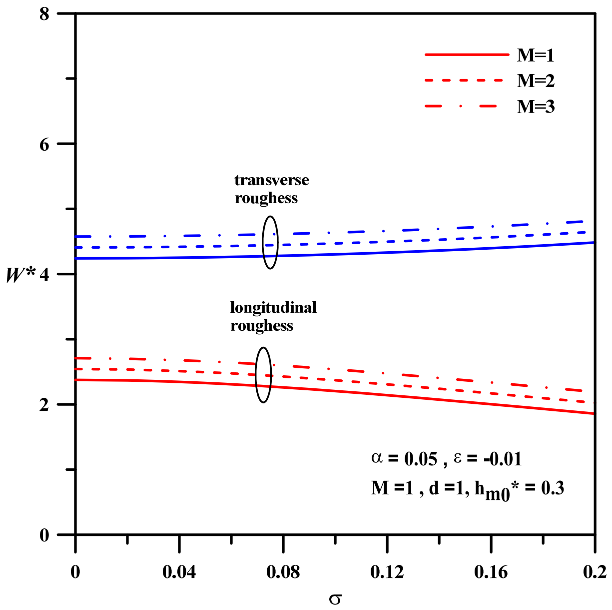

Fig. 5 Relation between W^* and σ with different M

The affections of stochastic random variable variance σ to the load carry capacity W^* subjected to an external magnetic strength M is depicted in Fig. 5. Apparently, transverse roughness has the effect of improving the load capacity, while the longitudinal one depressed. The stronger the external magnetic strength M is, the more increment of the load capacity the bearing system has. As considering the effect of stochastic variance of surface roughness, the load capacity of the transverse one dimensional type increases as the stochastic variance increase, while the longitudinal one has reverse trend, that is, load capacity decrease as stochastic variance increase.

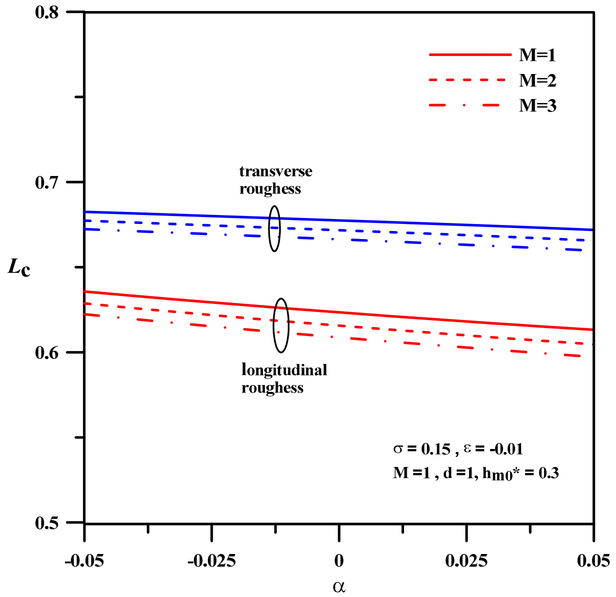

Fig. 6 Relation between L_c, and α with different M

Subjected to an external magnetic strength M, the position of non-dimensional center of pressure Lc varies with stochastic non-zero mean α is sketched in Fig. 6. As one can see from Fig. 2, if the stochastic roughness non-zero mean α increases, the position where the maximal film pressure P^*_{max} occurred will shift toward to the inlet zone, because of this, the center of pressure L_c of bearing system will shift slightly towards X=0.5. Considering the effect of surface roughness type to the center of pressure, as the maximal pressure occurrence of longitudinal one occurred in the left end of transverse one, thus, the center of pressure of transverse one dimensional surface roughness tilted bearing is closer to the outlet zone than that of longitudinal one. In addition, as the magnetic strength M increases, there is a decrement of the value L_c, namely, increasing the external magnetic strength, the center of pressure of bearing will shift further towards the inlet zone.

Subjected to an external magnetic strength M, the position of center of pressure Lc varies with variance σ is plotted in Fig. 7. If the surface roughness parameter σ increase in machine forming or finishing, the center of pressure of the longitudinal one dimensional type will shift towards to the geometry center of the tilted bearing. On the contrary, the transverse one has the reverse tendency, or shift towards to the outlet zone, but the effect is marginal.

Subjected to an external magnetic strength M, the squeezing film thickness h^*_m as function of the responding time T is drawn in Fig. 8. As shown in Fig. 8, the external magnetic strength M has the effect to prolong the responding time of bearing system, that is to say, the lubrication system of tilted bearing subjected to external magnetic strength can retard the upper plane to collide the bearing, therefore, the bearing system can sustain a longer time. Similarly, the transverse one dimensional roughness type has the larger responding time than that of longitudinal one operating at the same condition, thus, tilted bearing with transverse one dimensional type of surface roughness can prolong a working-hour than longitudinal one.

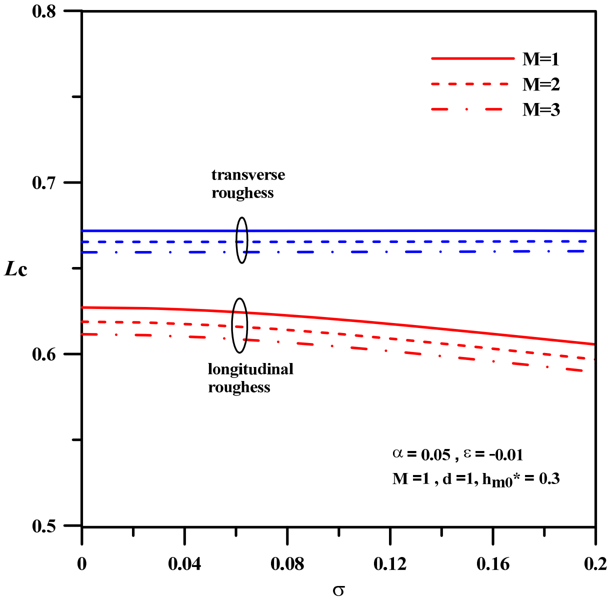

Fig. 7 Relation between L_c, and σ with different M

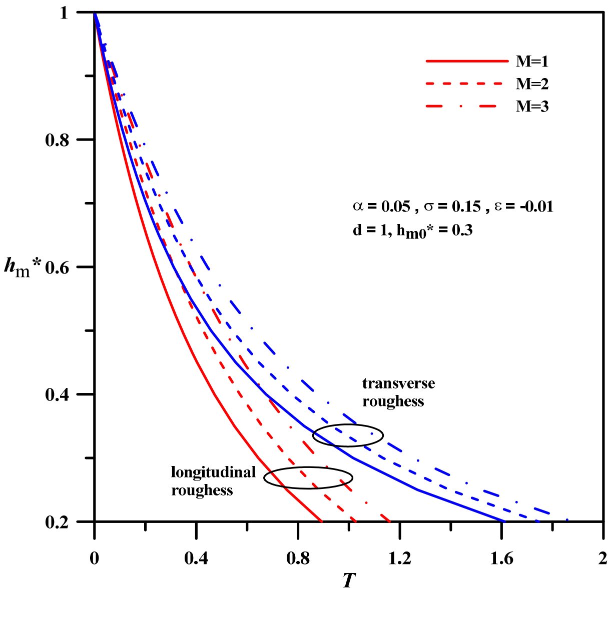

Fig. 8 Film thickness h_m^* as function of T with different M

IV. CONCLUSIONS

1. Comparing with that of the smooth bearing, the built-up pressure with longitudinal one dimensional roughness will decrease, and the position of maximal pressure shifts towards the outlet zone, while the transverse one has reverse trend.
2. When the tilted bearing system subjected to an external magnetic strength M, the built-up film pressure P^* can be improved. When the magnetic strength applied increases, the position where the maximal film pressure P^*_{max} occurred will shift towards the inlet zone.
3. The load capacity of the transverse one dimensional type increases as the stochastic variance σ increase, while the longitudinal one has reverse trend.
4. Increasing the external magnetic strength, the center of pressure L_c of bearing will shift further towards the inlet zone.
5. The transverse one dimensional roughness type has the larger responding time T than that of longitudinal one operating at the same condition

REFERENCES

[1] N. B. Naduvinamani, "Non-Newtonian effects of second-order fluids on double-layered porous Rayleigh-step bearings," *Fluid Dynamics Research*, 21, (1997) 495-507

[2] B. C. Majumdar, Introduction to Tribology of Bearing, Wheeler Pub, (1999)87-90.

[3] R. C. , M. V. Bhat, "Analysis of a Porous Exponential Slider Bearing Lubricated with a Ferrofluid Considering Slip Velocity," *J. of the Braz. Soc. of Mech. Sci. & Eng.*, 25(3), (2003) 264-268

[4] R. C. Shah, "Ferrofluid lubrication in step bearing with two steps," *Indus. Lub. and Tribo.*, 55(6), (2003) 265-267.

[5] M. Dobrica, , M. Fillon, "Reynolds' Model Suitability in Simulating Rayleigh Step Bearing Thermohydrodynamic Problems," *Tribo. Trans.*, 48, (2005) 522-530

[6] H. Christensen, "Stochastic models for hydrodynamic lubrication of rough surfaces," *Proc. Instn. Mech. Engrs.*, 55, (1969) 1013-1025.

[7] H. Christensen, K. Tonder, "The hydrodynamic lubrication of rough bearing surfaces of finite width," *ASME J. of Lub. Tech.*, 93, (1971) 324-330.

[8] K. Gururajan,, J. Prakash, "Surface roughness effects in infinitely long porous journal bearings," *J. of Tribo.*, 121, (1999) 139-146.

[9] H. L. Chiang,, T. L. Chou, C. H. Hsu, J. R Lin, "Surface roughness effects on the dynamic characteristics of finite slider bearings," *J. of C.C.I.T.*, 34(2), (2005) 29-39.

[10] P. I. Andharia, F. L. Gupta, G. M. Deheri, "On the shape of the lubricant film for the optimum performance of a longitudinal rough slider bearing," *Indus. Lub. and Tribo.*, 52(6), (2000) 273-276.

[11] G. M. Deheri, P. I. Dandharia, R. M. Patel, "Longitudinal rough slider bearing with squeeze film formed by a magnetic fluid," *Indus. Lub. and Tribo.*, 56(3), (2004) 177-187.

[12] N. B. Naduvinamani, A. Siddangouda, "Effect of surface roughness on the hydrodynamic lubrication of porous step-slider bearings with couple stress fluids," *Tribo. Int.*, 40, (2007) 780-793.