An Efficient Computational Method for Dynamic Interaction of High Speed Train and Railway Structure Including Derailment During An Earthquake

M. Tanabe[1], H. Wakui[2], M. Sogabe[2], N. Matsumoto[2] and Y. Tanabe[3]

[1] Kanagawa Institute of Technology, Atsugi, Kanagawa, 243-0292, Japan,

[2] Railway Technical Research Institute,Kokubunji, Tokyo, 185-8540, Japan

[3] Laboratory for Computational Mechanics Inc.,3-14-18 Kurihara-chuo, Zama, 252-0014, Japan

E-mail of corresponding author: tanabe@sd.kanagawa-it.ac.jp

Abstract – An efficient computational method to solve for the dynamic interaction between a high-speed train and the railway structure including derailment during an earthquake is given. The motion of the train is expressed in multibody dynamics. Efficient mechanical models to express contact-impact behaviors between wheel and the track structure including derailment during an earthquake are given. Rail and track elements with multibody dynamics and FEM combined are described. The motion of a railway structure is modelled with various finite elements and also with rail and track elements. A modal reduction is applied to solve the problem effectively. An exact time integration scheme has been developed that is free from a round-off error for very small time increments needed to solve the interaction between wheel and railway structure including derailment during an earthquake. Numerical examples are demonstrated.

Keywords - Dynamic interaction between a high-speed train and railway structure, Multibody dynamics, FEM, Derailment, Earthquake

I. INTRODUCTION

There is a very complicated dynamic interaction between a high-speed train and railway structure during an earthquake. The interaction between the train and railway structure during a strong earthquake creates a radical dynamic behaviors such as the huge impact of wheel on the rail, lifting of wheel, derailing, touching down and the impact on the track surface which leads to the high-frequency response higher than 1 k Hz mixed with the low-frequency motion of railway structure and train, that is so called a multiscale phenomenon of the frequency. It is very important to study the dynamic interaction between the train and railway structure including derailment behavior during an earthquake to build an earthquake-safe railway system.

Computational methods to solve for the dynamic interaction of a high-speed train and railway structure have been developed based on mutibody dynamics together with finite element method by many authors, and various mechanical models for the car, the railway structure and the interaction have been given depending on dynamic behaviors of interests and purposes to design railway systems [1-3]. However, very little work related to derailment and post-dearilment behaviors of a whole high-speed train on the railway structure during an earthquake has been reported so far.

In this paper, a simple and efficient computational method to solve for the dynamic interaction of a Shinkansen train (high-speed train in Japan) and railway structure including derailment behavior during an earthquake is given.

The motion of the train is modeled in multibody dynamics with nonlinear springs and dampers to connect all the components. Efficient mechanical models to express contact-impact behaviors between wheel and rail before derailment and between wheel and the track structure after derailment are given to solve the interaction between wheel and track structure effectively. Rail and track elements have been developed based on multibody dynamics and finite element method combined to solve the interaction between wheel and long railway components effectively.

The motion of railway structure is modeled with various finite elements such as truss, beam, shell, solid, and nonlinear spring and damper elements, and also with rail and track elements. The nonlinear dynamic response during an earthquake is obtained by solving equations of motions of the train and railway structure subjected to interactions between wheel and the track structure including derailment behavior. A modal reduction to the equations of motions is made to solve the large-scale problems effectively. The response calculation for the nonlinear equations of the train and railway structure during an earthquake requires very small time increments due to the high-frequency impact behavior between wheel and railway structure. An exact time integration scheme has been developed that is stable without a round-off error for very small time increments needed to solve the radical interaction between wheel and railway structure during an earthquake.

Based on the present method a computer program, DIASTARS, has been developed for the simulation of a Shinkansen train running at high speed on the railway structure including derailment during an earthquake. Numerical examples are demonstrated.

II. MECHANICAL MODEL OF A SHINKANSEN TRAIN

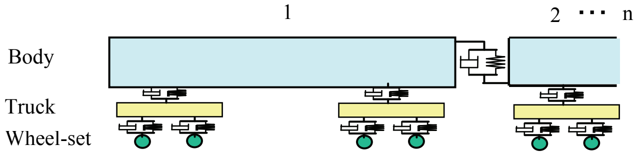


Fig. 1 Mechanical model of a Shinkansen train

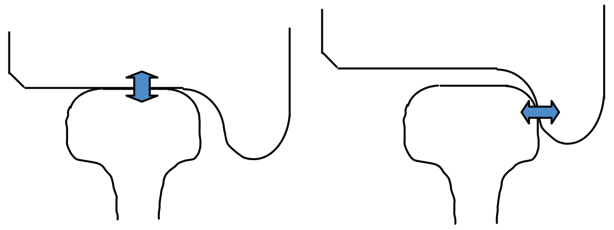
A Shinkansen train is modeled with rigid components of car body, truck and wheel-set connected by nonlinear springs and dampers as shown in Fig. 1. Assuming that the train runs at a constant speed, the equation of 3D motion of the train connected with n cars is derived with 31n degrees of freedom and written in a familiar matrix form as [4]

$$M^{v}\ddot{X}^{v} + D^{v}\dot{X}^{v} + K^{v}X^{v} = F^{v} \qquad (1)$$

where X^v and F^v are displacement and load vectors of the train, and M^v, D^v and K^v are the mass, damping, and stiffness matrices, respectively.

III. INTERACTION BETWEEN WHEEL AND TRACK

A. Contact between Wheel and Rail before Derailment: Assuming that the yawing and rolling of wheel-set are relatively small for the contact behavior between wheel and rail considered here, two dimensional geometries of the cross sections of wheel and rail are considered, and the contact-impact behavior in the normal direction on the contact surface between wheel and rail is modeled simply in two modes of the contact in the vertical and transverse directions as shown in Fig. 2.



(a) Vertical direction (b) Transverse direction

Fig. 2 Contact modes between wheel and rail

Assuming that wheel and rail are stiff enough, regarding the vertical mode of the contact the contact displacement between wheel and rail in the vertical direction, δ_Z, is expressed as a function of displacements of the rail z_R and the wheel z_w in the vertical direction and also the relative displacement on the contact surface between wheel and rail in the transverse direction d_y as follows

$$\delta_Z = \delta_Z(z_R, z_W, d_y) \qquad (2)$$

The contact displacement δ in the normal direction on the contact surface between wheel and rail is obtained by using the contact angle at the contact position. When wheel contacts on rail, there is a contact force created on the contact surface. The contact force on the contact surface between wheel and rail in the normal direction, H, is expressed as a function of δ and d_y

$$H = H(\delta, d_y) \qquad (3)$$

Regarding the transverse mode of the contact between wheel and rail, the contact displacement in the transverse direction δ_y is expressed as a function of d_y and δ_Z depending on the geometry of the cross sections of wheel and rail as

$$\delta_y = \delta_y(d_y, \delta_Z) \qquad (4)$$

When a wheel contacts on rail in the transverse direction, the contact force is also obtained in the same manner as the contact mode in the vertical direction described above.

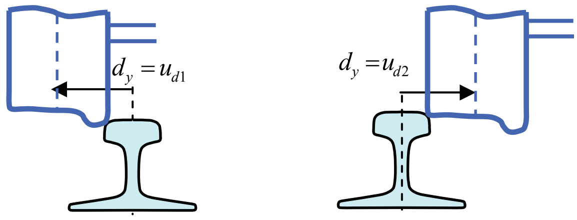
In the tangential and longitudinal directions on the contact surface between wheel and rail, constitutive equations to describe the relationship between creep forces and slipping rates of wheel are given [5]. The creep force in the tangential direction Q_c and yaw moment T_c due to the creep force in the longitudinal direction on the contact surface between wheel and rail are described mathematically here as functions of slipping rates of wheel in the longitudinal and tangential directions, S_x and S_t, and also of the spin rate around the normal vector on the contact surface, S_n, as [6]

$$Q_c = Q_c(S_x, S_t, S_n) \qquad (5)$$

$$T_c = T_c(S_x) \qquad (6)$$

When a wheel lifts on rail, there is no impact and creep forces created between the wheel and the rail.

B. Derailment Criterion: When the relative displacement between wheel and rail in the transverse direction, d_y, exceeds derailment criterions u_{d1} or u_{d2}, it is detected that the derailment in the field side or gauge side is initiated, respectively as shown in Fig. 3 that leads to post-derailment behaviors of the wheel on the track structure. Once derailment of a wheel occurs, it is assumed here that the wheel never returns to running on the rail. After derailment of wheel from rail during an earthquake the wheel touches down on the track structure and there is the interaction between wheel and the track structure.



(a) Field-side derailment (b) Gauge-side derailment

Fig. 3 Derailment criterion of left wheel

C. Contact between Wheel and Track Structure in the Vertical Direction after Derailment: After touching down of a wheel on the track structure, the relative displacement between the wheel and the track structure in the vertical direction, δ_{TZ}, after derailment is obtained as a function of displacements of the track structure z_T and the wheel flange z_F in the vertical direction depending on the geometries of the wheel, rail and track surface (Fig. 4) as

$$\delta_{TZ} = \delta_{TZ}(z_T, z_F) \qquad (7)$$

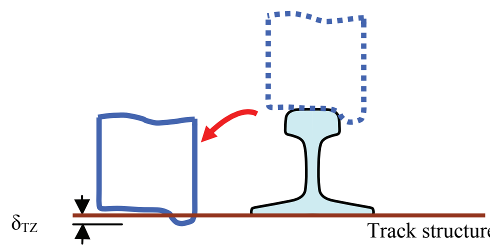


Fig. 4 Vertical relative displacement between wheel and track structure δ_{TZ}

When a wheel touches down on the track surface, the impact force of the wheel on the track structure on the contact surface, Q_{TZ}, is given here as a function of δ_{TZ} depending on material properties of the contact between the wheel and the track structure as

$$Q_{TZ} = Q_{TZ}(\delta_{TZ}) \tag{8}$$

D. Contact between Wheel and Guard after Derailment: Guards are attached on the track structure to prevent wheel deviating from the track after derailment during an earthquake to build an earthquake-safe railway system as shown in Fig. 5. After the derailment of a wheel during an earthquake, it contacts and impacts on the guard of the track structure in the transverse direction. The contact force Q_{Gy} is obtained here as a function of the embedded area between the wheel and the guard in the transverse direction as shown in Fig. 5, A_{Gy}, as

$$Q_{Gy} = Q_{Gy}(A_{Gy}) \tag{9}$$

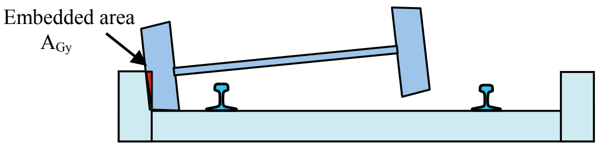
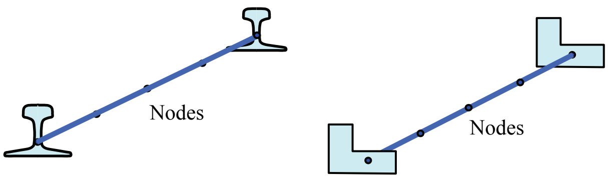


Fig. 5 Contact-impact between wheel and guard on the track structure in the transverse direction

If the guard has an enough height and strength for the impact force between wheel and guard, the wheel is guided well between left and right guards along the rail even after the derailment preventing wheel deviating from the track during an earthquake.

IV. MECHANICAL MODEL OF RAILWAY STRUCTURE

A. Rail and Track Elements: Long railway components in the rail direction such as rail and track are considered to move as rigid bodies of the motion in plane of the cross-section. Rail and track elements have been developed to solve contact-impact behaviors between wheel and rail in the pre-derailment and between wheel and the track structure in the post-derailment effectively for the actual railway structure. Fig. 6 shows rail and track elements where motions in plane of cross-sections are expressed by multibody dynamics (MD) and motions in out-of-plane are expressed by beam elements given along the rail direction. The elements mixed with MD and FEM are very effective to solve contact-impact problems between wheel and track structure including derailment with a small numbers of DOFs.



(a) Rail element (b) Track element

Fig. 6 Rail and track elements

B. Equation of Motion of a Railway Structure: A railway structure is modeled with various finite elements such as truss, beam, shell, solid, mass, and nonlinear spring and damper elements, and also with rail and track elements for long railway components depending on the mechanical behavior. Assembling all elements in the model, the whole equation of motion of a railway structure is written in a familiar matrix form as

$$\mathrm{M}^b\ddot{\mathrm{X}}^b + \mathrm{D}^b\dot{\mathrm{X}}^b + \mathrm{K}^b\mathrm{X}^b = \mathrm{F}^b \tag{10}$$

where X^b and F^b are the displacement and load vectors, and M^b, D^b, and K^b are the mass, damping and stiffness matrices of the railway structure, respectively. However, note that F^b includes all nonlinear forces, and is a nonlinear function of X^b, $\dot{X}^b$, X^v and $\dot{X}^v$.

V. NUMERICAL METHOD

A. Modal Reduction: A modal transformation is applied to displacement vectors of the train and the railway structure to solve for the large-scale problems during an earthquake effectively as

$$X^v = \Phi^v Z^v \tag{11}$$
$$X^b = \Phi^b Z^b \tag{12}$$

where Φ^v and Φ^b are rectangular matrices made of the mode vectors of the train and railway structure, and Z^v and Z^b are the modal coordinates, respectively. Moving nonlinear terms in (1) and (10) to the right sides, and operating the modal transformation by Φ^v and Φ^b respectively, equations of motions of the train and the railway structure are derived in the modal coordinates as

$$\ddot{Z}^V + \tilde{C}^V\dot{Z}^V + [(\omega_i^V)^2]Z^V = \tilde{F}^V \tag{13}$$
$$\ddot{Z}^B + \tilde{C}^B\dot{Z}^B + [(\omega_i^B)^2]Z^B = \tilde{F}^B \tag{14}$$

where ω_i^V and ω_i^B are angular frequencies of i-th mode in the train and railway structure, respectively, and $[(\omega_i^V)^2]$ denote a diagonal matrix with i-th diagonal element of $(\omega_i^V)^2$. However

$$\tilde{C}^V = (\Phi^V)^T C^V \Phi^V \tag{15}$$
$$\tilde{C}^B = (\Phi^B)^T C^B \Phi^B \tag{16}$$
$$\tilde{F}^V = (\Phi^V)^T F^V \tag{17}$$
$$\tilde{F}^B = (\Phi^B)^T F^B \tag{18}$$

where the superscript T denotes the transpose of a matrix. Note that the nonlinear terms for the train and railway structure are included in F^V in (17) and F^B in (18), respectively.

B. Exact Time Integration: Equations of motions of the train and the structure are solved in the modal coordinates for each time increment by the exact time integration scheme by approximating the right side terms with linear functions and applying the exact time integration for each small time increment as the numerical time integration may cause a round-off error for very small time increments needed to solve the radical dynamic interaction including derailment during an earthquake. However, since the equations are strongly nonlinear, iterative calculations are needed during each time increment until the unbalanced force between the train and railway structure becomes small enough within a tolerance specified.

Based on the present method, a computer program DIASTARS has been developed for the simulation of a Shinkansen train at high speed on the railway structure including derailment during an earthquake.

VI. NUMERICAL EXAMPLES

A. One Dimensional Nonlinear Dynamic Problem: The exact time integration scheme mentioned in the previous section is applied to one dimensional nonlinear dynamic problem with a contact element attached as shown in Fig. 7, where ω is the natural frequency, g the gravity force of a unit mass, h is a damping constant and H is the impact force in the contact element with the initial gap G and the contact stiffness K.

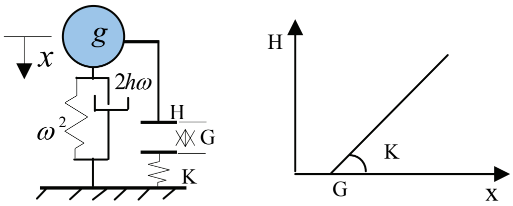


Fig. 7 One dimensional nonlinear dynamic problem

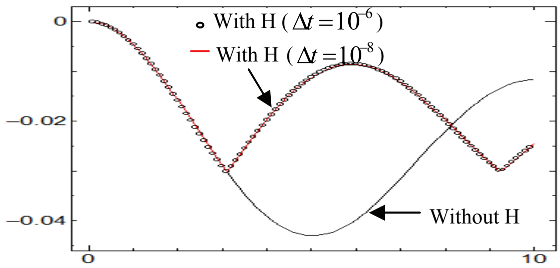


Fig. 8 Displacement response by the exact time integration

For the problem of $\omega = 2\pi/10$ rad/sec, h=0.1, $K = 5\times10^3$ N/m, G=0.03m and g=9.8N, the response was obtained by the exact time integration. Fig. 8 shows the displacement response for very small time increments of $\Delta t = 10^{-6}$ and 10^{-8} for the case with H (contact force) and also for the case without H. For the very small time increments, the exact time integration gives the exact solution. On the other hand, the numerical time integration by Newmark method failed due to the round-off error for both cases with H and without H for very small time increments of both $\Delta t = 10^{-6}$ and 10^{-8}.

B. Simulation of a Shinkansen Train Running on a Four-Spanned Bridge During an Earthquake: Fig. 9 shows a four-spanned steel-concrete hybrid bridge as long as 320m which is made of steel box girders and the concrete slab. The simulation of a Shinkansen train with 8 cars connected running at a speed of 260km/h on the bridge during an earthquake has been conducted. An artificial earthquake wave L1 used in the Japanese design standard for railway structure with the maximum acceleration of 1.95 m/sec^2 was given at bases of all piers of the bridge in the transverse direction. The rail and the steel box girder were modelled with beam elements. All piers were modelled with nonlinear spring elements, where a trilinear elastic-plastic material model with a kinematic hardening is employed. It has a large-scale model of 5,744 elements and 24,716 DOFs including that of the train connected with 8 cars. Fig. 10 shows the transverse acceleration response of the pier P3 at the top and track positions. The maximum acceleration of the track at P3 is shown to be about 6 m/sec^2. Fig. 11 shows transverse and roll accelerations of the car body at the 1st car. The maximum transverse acceleration is shown to be about 7 m/sec^2 during the earthquake. The maximum lifting height of wheel was about 25mm at the 6th car, however there was no derailment detected of all wheels during the earthquake.

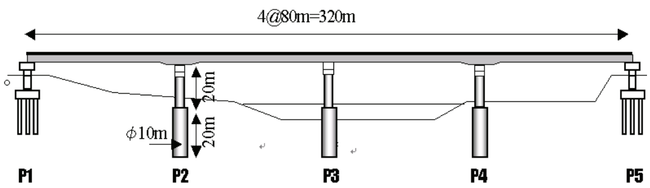


Fig. 9 A Shinkansen train with 8 cars running at a speed of 260km/h on a four-spanned bridge during an earthquake

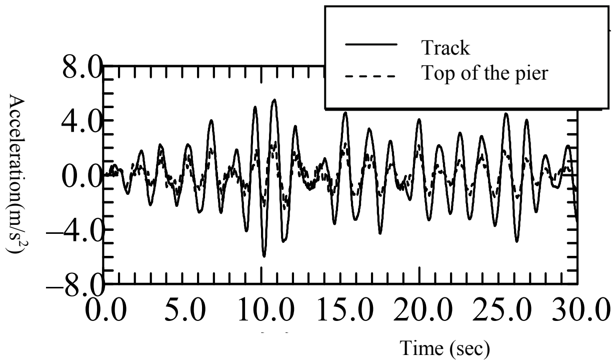


Fig. 10 Transverse acceleration response of pier P3

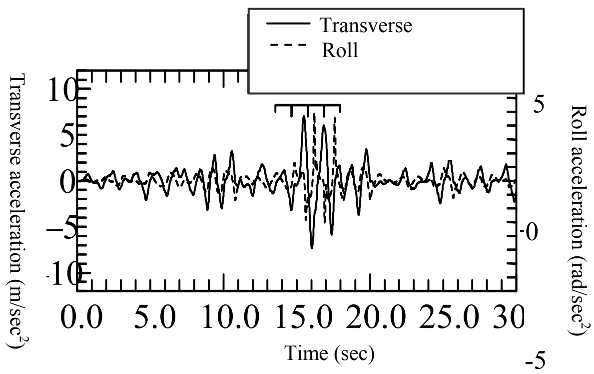


Fig. 11 Transverse and roll accelerations of the car body at the 1st car

C. Simulation of a Shinkansen Car Running on the Viaduct with Guards Attached during an Earthquake: The simulation of a Shinkansen car running at a speed of 275 km/h on the ladder track with guards attached on the three spanned viaduct with the height of 10m, the span-length of 8m and the width of 11.6m during an earthquake as shown in Fig. 12 has been conducted. The ladder track is a new type of the track, where ladder-shaped composite concrete beams to support rails are tied with steel pipes and guards are attached to prevent wheel deviating from the track even after derailment during a strong earthquake [7].

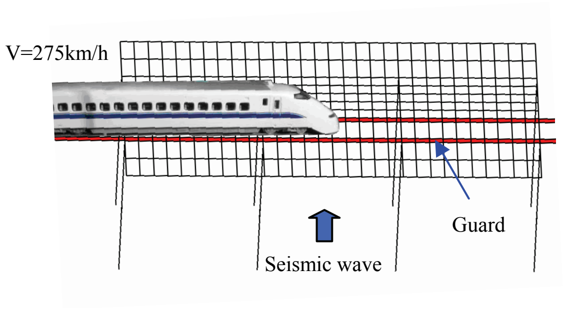


Fig. 12 A Shinkansen car on the ladder track with guard

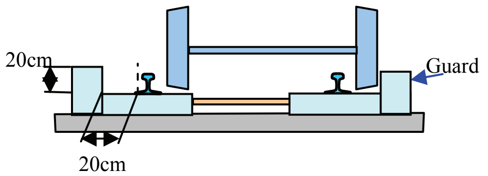


Fig. 13 Cross section of the ladder track with guard attached

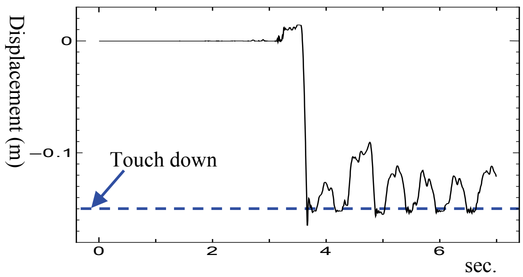


Fig. 14 Vertical displacement of the right wheel of the 1st wheel-set

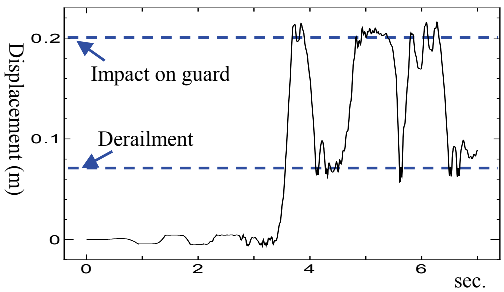


Fig. 15 Relative displacement between the right wheel and rail in the transverse direction

A sinusoidal seismic wave with the maximum acceleration of 3.8 m/sec^2, the wave number of five, and the frequency of 1Hz was given from the base in the transverse direction. The frame structure of the viaduct is modeled with beam elements, and the concrete slab with shell elements. The rail and the track with guards attached on the concrete slab was modeled with rail and track elements, respectively.

Fig. 14 shows the vertical displacement response of the right wheel at the 1st wheel-set. It is shown that the wheel runs onto the rail about 2cm in height, derails, touches down on the track, and lifts on the track several times during the earthquake. Fig. 15 shows the relative displacement response between right wheel and rail in the transverse direction, δ_y , at the 1st wheel-set. When δ_y exceeds u_{d1} (7cm), the derailment to the field side is initiated, the wheel touches down on the ladder track, impact on the guard of the track in the transverse direction, rebounds and gets back to the rail, and moves between the rail and the guard without deviating from the track. The track with the guards attached was shown to be effective to prevent the wheel deviating from the track after derailment during the seismic wave.

VII. CONCLUSIONS

A simple and efficient computational method to solve the combined dynamic response of a high-speed train and railway structure including derailment during an earthquake was given. Efficient mechanical models to solve contact-impact behaviors between wheel and the track structure including derailment were given. Modal reduction was applied to equations of motions of train and railway structures to solve for the large-scale problems effectively. The exact time integration scheme has been developed to avoid numerical error for very small time increments needed to solve the radical dynamic interaction between wheel and track structure during an earthquake. Some examples including derailment during an earthquake were demonstrated. The computational method developed here would be effective to design an earthquake-safe railway system.

ACKNOWLEDGMENT

The authors would like to thank JSPS (Japan Society of the Promotion of Science) for the financial support of part of this work.

REFERENCES

[1] M. Tanabe, H. Wakui and N. Matsumoto, “The finite element analysis of dynamic interactions of high-speed Shinkansen, rail, and bridge”, *Computers in Engineering,* ASME Book No. G0813A, (1993)17-22.
[2] A. Jaschinski, G. Schupp and H. Netter, “Demonstration of simulation potentials in railway vehicle system dynamics through selected examples”, Proceedings of World Congress on Railway Research, vol. D, (1997)15-23.
[3] S. Bruni, A. Collina, R. Corradi and G. Diana, “Numerical simulation of train-track-structure interaction for high speed railway systems”, Proceedings of Structures for High-Speed Railway Transportation, IABSE Symposium, Antwerp, Belgium, (2003).
[4] M. Tanabe, S. Komiya, H. Wakui, N. Matsumoto and M. Sogabe, “Simulation and visualization of a high-speed Shinkansen train on the railway structure”, *Japan J. Indust. Appl. Math.* vol.17, (2000)309-320.
[5] J. J. Kalker, *Three-dimensional elastic bodies in rolling contact,* Kluwer, (1990).
[6] *Dynamics of railway vehicle,* JSME, (1994).
[7] K. Asanuma, M. Sogabe, T. Watanabe, J. Okayama and H. Wakui, “Development of a Ballasted ladder track equipped with a vehicle guide device”, *RTRI Report* Vol.23, No.2, (2009)27-32.