Modeling of a Time Dependent Alternative Vehicle Routing Problem with Time Windows

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Abstract—In this study, we consider congested situation and edge choosing problem simultaneously in the VRP logistics and this problem is called the Time Dependent Alternative Vehicle Routing Problem with Time Windows (TDAVRPTW). Given a multi-graph, the problem can be formulated into a mixed integer programming model with continuous dependent travel time functions. Numerical example shows the applicability of the model from the viewpoints of cost and computation efficiency.

Keywords-TDAVRPTW; Time Dependent; Multi-Graph; Edge Choosing; PSO-based Alorithm

I. INTRODUCTION

Consider a delivery problem for a distribution center or a cargo company. In different time periods, a vehicle traveling situation on the edges changes with the time of a day and there are peak-hour problems in the congestion environment. When vehicle travels on the same edges but at different time, its travel time is different. If the cargo company or vehicle drivers need to plan the delivery routes within a time pressure, they probably will choose the other edges with less traffic to save travel time from congestion. This happens especially in the peak hour.

The formal research tackled such problem by considering only one edge between each pair of customers, which apparently, are not suitable for this situation. In addition, in order to minimize the total logistic cost for a company, in addition to the number of vehicles to be dispatched, balance between travel time and distance has to be considered.

As a result, except for the basic considerations in the Time Dependent Vehicle Routing Problem (TDVRP), there are other properties for this kind of problem that we should take into account. First, the service order for each vehicle should be determined under a situation that there are two edges between each pair of customers. Second, different weights of importance embedded in the different costs can be evaluated according to their management decisions.

In conclusion, the Time Dependent Alternative Vehicle Routing Problem with Time Windows (TD-AVRP-TW) is defined as follows：

"A fleet of vehicles with fixed capacities are assigned to a number of customers whose service requests of demands and time windows are given in a multi-graph network, the problem is to find an optimal route for each vehicle, which departs from a depot, serves the customers and returns to the original depot, such that the total aggregated cost including fixed cost, time cost and distance cost is minimized."

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II. LITERATURE REVIEW

Since the TDAVRPTW is an extension of the TDVRP, we are going to review some literatures that focused on the TDVRP.

The abbreviation "TDVRP" doesn't include the words "TW (time windows) ", however the problems that the authors dealt with in the literatures took the time windows into account.

The TDVRP (or TDVRPTW) can be described as follows according to Malandraki *et al.* [5]：

The purpose of the Time Dependent VRP analysis is to determine a set of vehicle routes originating and terminating at a single depot such that with the minimum total travel cost, all customers are visited within their available time windows exactly once, and the total demand of the customers assigned to each route does not violate the capacity of the vehicle and the travel time of vehicle is dependent on the time of the day, that is, it will change with the different time periods. We concluded three properties of TDVRP in the literatures：

First, in terms of cost, most of studies in the literatures only consider the travel time cost in objective functions [1][2][5].

Second, consider the time windows, besides Ichoua *et al.* [3] considered soft time windows, most of the papers adopted hard time windows.

Third, Malandraki *et al.* [5] used the discrete travel time functions to describe the relations between the time of the day and the travel time. After Ichoua *et al.*[3] presented the continuous travel time functions in the TDVRP, most of the references[1][2][4] used continuous travel time functions in the following studies. Figure 1 is the travel speed distribution (a) and its corresponding travel time distribution (b) derived from the speed distribution by integration.

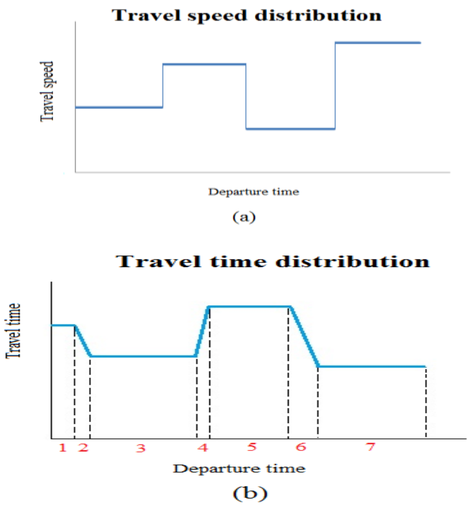

Figure 1：Travel speed and time distribution

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III. MODELLING OF TDAVRPTW

The problem that we deal with is presented. First the properties of the TDAVRPTW will be introduced in section 3.1, and then a mixed integer programming model is formulated in section 3.2. Finally model evaluation is provided in section 3.3.

A. Properties of TDAVRPTW

Assume that at most two edges between each pair of customer nodes with the properties that one with shorter distance is the *designed edge* of which the travel speed varies with respective to the travel time of the day; and the other one with longer distance is called *alternative edge* of which the travel speed is constant. Figure 2 is a sketch diagram of designed edge and alternative edge between node A and B.

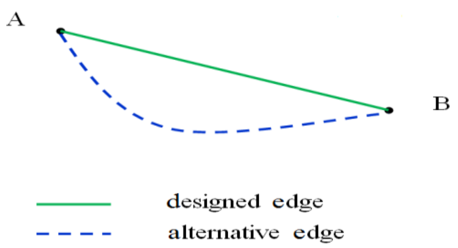

Figure 2：designed edge and alternate edge

The other properties of the TDAVRPTW are described as follows：

First, in the procedure of doing the transportation planning, there are many costs should be taken into account. We will consider aggregated weights of these costs instead of single cost.

Second, we allow the delivery driver arrives before the customer's earliest service time window; however, the vehicle must wait until the earliest service time is opened with a waiting penalty. But the vehicle cannot arrive after the latest window time constraint. This kind of restriction is reasonable because the cargo company should always consider the customers as a priority.

Finally, since continuous travel time function satisfies the FIFO (First In, First Out) principle, it is more suitable to be adopted in our study.

B. Model Formulation

The TDAVRPTW can be represented with a complete undirected *multi-graph* G(N, E), where N is the set of nodes, N={0,1, 2, …, n, n+1}, where 0 is the distribution center and the collection center is denoted by n+1 (they can be the same point). E is a set of edges connecting pairs of nodes, $E = E_1 \cup E_2$. E_1 is the edge set of designed edges and E_2 is the edge set of alternative edges. V is the set of vehicles, V ={1, 2, …, v_{max}} where v_{max} is the maximum available number of vehicles. Finally, M is the number of time intervals considered for each designed edge, M ={1, 2, …, m_{max}} and m_{max} is the maximum index of time interval.

Minimize $$\sum_{\sigma=1}^{3}\alpha_\sigma\lambda_\sigma Z_\sigma \qquad (1)$$

$$Z_1 = F\sum_{j=1}^{n}\sum_{v}\Big(\sum_{m} x_{0jv}^{m} + y_{0jv}\Big)$$

$$Z_2 = TC(r_{n+1} - l_0) + P\sum_{i=1}^{n} w_i$$

$$Z_3 = \sum_{(i,j)\in E}\sum_{v}\Big(\sum_{m} DO_{ij}x_{ijv}^{m} + DA_{ij}y_{ijv}\Big)$$

Subject to

$$\sum_{m} x_{ijv}^{m} + y_{ijv} \le 1 \qquad \forall (i,j)\in E,\ \forall v\in V \qquad (2)$$

$$\sum_{j=1}^{n+1}\sum_{v}\Big(\sum_{m} x_{ijv}^{m} + y_{ijv}\Big) = 1 \qquad \forall i\in N \qquad (3)$$

$$\sum_{i=0}^{n}\sum_{v}\Big(\sum_{m} x_{ijv}^{m} + y_{ijv}\Big) = 1 \qquad \forall j\in N \qquad (4)$$

$$\sum_{j=1}^{n}\Big(\sum_{m} x_{0jv}^{m} + y_{0jv}\Big) \le 1 \qquad \forall v\in V \qquad (5)$$

$$\sum_{i=0}^{n}\Big(\sum_{m} x_{ihv}^{m} + y_{ihv}\Big) - \sum_{j=1}^{n+1}\Big(\sum_{m} x_{hjv}^{m} + y_{hjv}\Big) = 0 \qquad \forall h\in N,\ \forall v\in V \qquad (6)$$

$$\sum_{v}\Big(\sum_{m} x_{0,n+1,v}^{m} + y_{0,n+1,v}\Big) = 0 \qquad (7)$$

$$\sum_{i=1}^{n} D_i\sum_{j=1}^{n+1}\Big(\sum_{m} x_{ijv}^{m} + y_{ijv}\Big) \le Q_v \qquad \forall v\in V \qquad (8)$$

$$(A_i + S_i)\Big[\sum_{j=1}^{n+1}\Big(\sum_{m} x_{ijv}^{m} + y_{ijv}\Big)\Big] \le l_{iv} \qquad \forall i\in N\cup\{0\},\ \forall v\in V \qquad (9)$$

$$l_{iv} \le (B_i + S_i)\Big[\sum_{j=1}^{n+1}\Big(\sum_{m} x_{ijv}^{m} + y_{ijv}\Big)\Big] \qquad \forall i\in N\cup\{0\},\ \forall v\in V \qquad (10)$$

$$t_{ijv}^{m} \geq K_{1ij}^{m} l_{iv} + K_{2ij}^{m} \quad \forall (i,j) \in E, \forall v \in V, \forall m \in M \quad (11)$$

$$l_{jv} - l_{iv} + M_1(1 - x_{ijv}^{m}) \geq t_{ijv}^{m} + S_j \quad \forall (i,j) \in E, \forall v \in V, \forall m \in M \quad (12)$$

$$l_{jv} - l_{iv} + M_1(1 - y_{ijv}) \geq T_{aij} + S_j \quad \forall (i,j) \in E, \forall v \in V, \forall m \in M \quad (13)$$

$$l_{iv} + M_2 x_{ijv}^{m} \leq T_{ij}^{m} + M_2 \quad \forall (i,j) \in E, \forall v \in V, \forall m \in M \quad (14)$$

$$l_{iv} - T_{ij}^{m-1} x_{ijv}^{m} \geq 0 \quad \forall (i,j) \in E, \forall v \in V, \forall m \in M \quad (15)$$

$$l_i = \sum_{v} l_{iv} \quad \forall i \in N \cup \{0, n+1\} \quad (16)$$

$$r_i = l_i - S_i \quad \forall i \in N \cup \{n+1\} \quad (17)$$

$$w_i \geq A_i - r_i \quad \forall i \in N \quad (18)$$

$$x_{ijv}^{m}, y_{ijv} \in \{0,1\} \quad \forall (i,j) \in E, \forall v \in V, \forall m \in M \quad (19)$$

$$l_{ik} \geq 0 \quad \forall i \in N \cup \{0, n+1\}, \forall v \in V \quad (20)$$

$$t_{ijv}^{m} \geq 0 \quad \forall (i,j) \in E, \forall v \in V, \forall m \in M \quad (21)$$

$$l_i, r_i, w_i \geq 0 \quad \forall i \in N \cup \{0, n+1\} \quad (22)$$

The objective function (1) shows that this model minimizes the total route cost, which consists of transportation fixed cost (Z_1), time cost (Z_2) and distance cost (Z_3). The decision maker can give different weights (α_σ) to decide the importance of each cost in the transportation planning. Where x_{ijv}^{m} and y_{ijv} are binary variables which represent whether the designed edge during time interval *m* or alternative edge is chosen or not if vehicle *v* departs node *i* toward node *j*. l_{iv} is the departure time of vehicle *v* from node *i*. r_i and w_i are arriving and waiting time at node *i*.

Constraint (2)~(7) are route constraints. Constraint (8) is the capacity constraint, it limits the total demand in one route excessive to the capacity of a vehicle. Constraint (9) and (10) are time window constraints, which indicate that the departure time from any customer node should fall in the departure time window. ($[A_i, B_i]$ is the time window at node *i*, $\forall i \in N \cup \{0, n+1\}$, S_i is the service time of customer *i*.)

Constraint (11)~(18) are time constraints where t_{ijv}^{m} and T_{aij} are the travel time of designed edge and alternative edge. Constraint (11) is the relation inequality of t_{ijv}^{m} and l_{iv}. Constraint (12) and (13) show that departure time from each node is at least equal to the sum of travel time from the preceding node and service time of current node when $x_{ijv}^{m} = 1$ or $y_{ijv} = 1$ (Only one of them will equal to 1). Constraint (14) and (15) ensure that the proper time interval *m* is chosen between nodes *i* and *j* according to the departure time from node *i*. Constraint (16) is the total sum of departure time from node *i*, where l_0 is the sum of the departure time from the distribution center of all vehicles. Constraint (17) computes the arriving time at node *i*. Constraint (18) shows the waiting time at node *i*

Constraint (20)~(22) are decision variable constraints. x_{ijv}^{m} and y_{ijv} are binary variables. The other variables are positive real numbers.

C. Model Evaluation

There are $5mn^2v$ constraints and $4mn^2v$ variables in this model where *m* is the number of time intervals considered for each designed edge, *n* is the number of customers and *v* is the number of vehicles. When *n* increases, the complexity of the problem will increase quickly.

Lemma 1: The cost in the TDAVRPTW will always be smaller than or equal to the TDVRPTW.

Numerical example has been done by adopting the travel speed functions which were generated by Donati *et al.* [1] for the designed edge. The results show the highly applicability of the model.

IV. *CONCLUSION AND DISCUSSION*

The TDAVRPTW that we propose in this paper take the edge choosing issue into account and offer the decision maker in the transportation planning to choose the cost weights combination that satisfies his or her needs. Furthermore, the planners can get the service information like departure or arriving time when they execute the model.

First we review some papers that were related to TDVRP, we find out using continuous travel time function can approximate the real situation more appropriate. In addition, we introduce the concept of alternative edge which is longer in distance and the travel time is a constant but if the service driver travels this edge, the driver can save the costs when the designed edge is congested.

Then a mixed integer programming model is developed with the discussion of its properties

Since TDAVRPTW is an NP hard problem, we have developed a heuristic method based on Particle Swarm Optimization to facilitate its large-scale applications. Brief description is as follows:

A. Two-stage Particle Swarm Optimization

Particle Swarm Optimization (PSO) is easy to implement, and the parameters which are needed to be adjusted are less than Genetic Algorithms (GA) (Tang *et al.*, 2011). PSO's fast convergence (Wang *et al.*, 2010) and good performances on the continuous search space have drawn our attention, and is adopted in this study.

In this section, the concept of the design of a heuristic algorithm based on PSO is first described in Section 4.1. Then, the procedure of implementing PSO is presented in Section B.

B. Design Concept of the Proposed PSO

In the solution to the TDAVRP, there are two main variables that must first be established: the service order for each vehicle; the departure time at each node and the selection of edge between each pair of nodes. It is difficult to consider these variables simultaneously, because the departure time of each node is based on the sequence of customers.

To facilitate a solution, this study proposes a two-stage PSO, including the Primary and Secondary PSOs. The Primary PSO is used to determine the service order for each vehicle and the Secondary PSO provides information relates to the travel strategies such as departure time and edge selection. Local improvement is also included to improve service sequences derived from the Primary PSO.

The procedure of the two-stage PSO is displayed in Figure 3. The algorithm begins with the Primary PSO, after decoding the particles into different routes and improving the quality of the solutions by local improvement. It then proceeds to the Secondary PSO and performed Secondary PSO operations. Finally, the best particles with the lowest objective value are used in the Secondary PSO to perform the Primary PSO operations. The stopping rules in these two PSOs are all the given number of iterations.

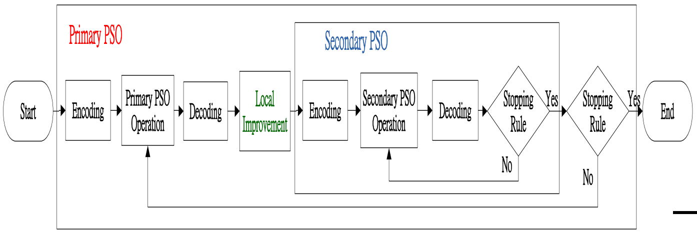

Figure 3 The procedure of the PSO Algorithm

C. Computability Analysis

By using the defined speed distribution functions, Table 1 shows the computational results of CPLEX and the PSO algorithm using Solomon's small scaled test problems R101 with 5 nodes, in which the values are the averages of the results of 10 test problems with different combinations of travel speed functions.

When the size of the problem increases to 25 nodes, CPLEX is unable to provide a feasible solution. Further tests have been done on the instances with 25, 50, and 100-nodes, respectively. The results have shown that the algorithm is able to obtain feasible solutions within 13 minutes when the problem size is smaller than 100 nodes.

From this preliminary experiment, the results have shown that the proposed PSO-based algorithm is promising on practical applications. Further study will be placed on the improvement of solution algorithm so that both accuracy and efficiency will reach to applicability standard.

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Table 1 Comparisons of CPLEX and the PSO in 5-node instance

Problem	ILOG CPLEX			PSO		
R101 (5nodes)	Min_ cost($)	Time(s)	Number of vehicles	Min_cost($) (deviation) [error rate]	Ave_time(s) [relative computation time]	Ave_ Vehicles
Average	106.23	11530	3	106.36 (0.13) [0.12%]	7.85 [0.07%]	3

Deviation = PSO Min_cost–CPLEX optimal.

Error rate = (PSO Min_cost−(CPLEX optimal solution)) / (CPLEX optimal solution).

Relative computation time = PSO Ave_time/CPLEX time.

Ave_Vehicles = the average number of vehicles (routes) in 5 running times.