Modelling of Damage Evolution in The Vicinity of Frictional Interfaces in Metal Forming

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Abstract - Conventional ductile fracture criteria are not applicable in the vicinity of maximum friction surfaces for several rigid plastic material models because the equivalent strain rate (second invariant of the strain rate tensor) approaches infinity near such surfaces. In the present paper, a non-local ductile fracture criterion generalizing the modified Cockroft-Latham ductile fracture criterion is proposed to overcome this difficulty with the use of conventional local ductile fracture criteria. The final form of the new ductile fracture criterion involves the strain rate intensity factor which is the coefficient of the principal singular term in a series expansion of the equivalent strain rate in the vicinity of maximum friction surfaces. When the velocity field is not singular, the new ductile fracture criterion reduces to the modified Cockroft-Latham criterion. The strain rate intensity factor cannot be found by means of commercial finite element packages since the corresponding velocity field is singular. In the present paper, the new fracture criterion is illustrated with the use of an approximate semi-analytical solution for plane strain drawing. It is shown that the prediction is in qualitative agreement with physical expectations.

Keywords - friction, singularity, ductile fracture, metal forming.

I. INTRODUCTION

Reviews of ductile fracture criteria are given in [1-3]. The Cockroft-Latham ductile fracture criterion [4] and its modifications are widely used in applications [5 – 12 among others]. These criteria, as well as many other ductile fracture criteria, involve the equivalent strain rate. On the other hand, the equivalent strain rate approaches infinity in the vicinity of maximum friction surfaces [13]. The definition for the maximum friction surface depends on the material model chosen. For example, the friction stress at sliding is equal to the shear yield stress of the material in the case of rigid perfectly plastic material. Ductile fracture sometimes occurs near frictional interfaces in metal forming processes [6]. The aforementioned behaviour of the equivalent strain rate in the vicinity of maximum friction surfaces is not compatible with ductile fracture criteria since they predict the fracture initiation at the very beginning of any process independently on other process conditions. A possible way to overcome this difficulty is to use non-local ductile fracture criteria. In the present paper such a criterion generalizing the modified Cockroft-Latham criterion [5] is proposed and then adopted to predict the fracture initiation in plane strain drawing. Other non-local ductile fracture criteria have been proposed in [14, 15].

II. NON-LOCAL DUCTILE FRACTURE CRITERION

The modified Cockroft-Latham ductile fracture criterion is given by [5]

$$\int_0^t \frac{\sigma_1}{\sigma_{eq}} \xi_{eq} dt = C_1 . \qquad (1)$$

Here σ_1 is the maximum principal stress, σ_{eq} is the equivalent stress, ξ_{eq} is the equivalent strain rate, t is the time, and C_1 is a material constant. The equivalent strain rate is defined by $\xi_{eq} = \sqrt{2/3}\sqrt{\xi_{ij}\xi_{ij}}$ where ξ_{ij} are the components of the strain rate tensor and the equivalent stress by $\sigma_{eq} = \sqrt{3/2}\sqrt{\tau_{ij}\tau_{ij}}$ where $\tau_{ij} = \sigma_{ij} - \sigma\delta_{ij}$, σ_{ij} are the components of the stress tensor, σ is the hydrostatic stress, and δ_{ij} is Kroneker's symbol. In the case of rigid perfectly plastic solids the equivalent strain rate approaches infinity in the vicinity of maximum friction surfaces according to the following rule [13]

$$\xi_{eq} = D/\sqrt{s} + o\left(1/\sqrt{s}\right), \quad s \to 0 \qquad (2)$$

where D is the strain rate intensity factor independent on s and s is the normal distance to the maximum friction surface. For this material model the maximum friction surface is defined by the condition

$$\tau_f = \tau_s \qquad (3)$$

where τ_f is the friction stress and τ_s is the shear yield stress. The condition (3) is valid in the case of sliding. The equivalent strain rate follows the rule (2) for several rigid plastic material models [16-19], though the formulation of the maximum friction law may differ from (3).

Substituting equation (2) into equation (1) shows that the fracture criterion is not applicable in the vicinity of maximum friction surfaces. In fact, it predicts the fracture initiation at the very beginning of any process independently on other process conditions. It is very similar to the mechanics of cracks where fracture conditions from the strength of materials cannot be used for cracked bodies because stress components approach infinity near the crack tip (for example, [20]). A possible approach to overcome this difficulty in the mechanics of cracks has been proposed in [21]. A similar approach is adopted in the present paper to make equation (2) compatible with the ductile fracture criterion (1). Note that even though the equivalent strain rate does not approach infinity in real processes, layers of intensive plastic deformation frequently appears near frictional interfaces in metal forming processes [22-25]. Equation (2) is in qualitative agreement with this experimental fact because it predicts the existence of a layer of intensive plastic deformation in the vicinity of maximum friction surfaces.

An average equivalent strain rate in the vicinity of maximum friction surfaces can be defined by

$$\Xi_{eq} = \frac{1}{s_c}\int_0^{s_c} \xi_{eq} ds, \tag{4}$$

where s_c is the thickness of the layer of intensive plastic deformation. A method for determining s_c has been proposed in [25]. Substituting equation (2) into equation (4) gives

$$\Xi_{eq} = 2D/\sqrt{s_c} \tag{5}$$

to leading order. The ductile fracture criterion (1) can be generalized by replacing ξ_{eq} with Ξ_{eq}. Then, with the use of equation (5),

$$2\int_0^t \frac{\sigma_1}{\sigma_{eq}}\frac{D}{\sqrt{s_c}}dt = C. \tag{6}$$

where C is a material constant whose value is in general different from the value of C_1. It is obvious from equation (4) that $\Xi_{eq} \equiv \xi_{eq}$ at points of the friction surface if the equivalent strain rate is described by a non-singular function and $s_c \to 0$. Therefore, the new non-local fracture criterion reduces to the modified Cockroft-Latham criterion in such cases.

In the case of stationary processes $dt = dl/u_\tau$ where dl is the infinitesimal arc length of streamlines coinciding with the maximum friction surface and u_τ is the velocity component tangent to the friction surface. Therefore, for stationary processes equation (6) transforms to

$$2\int_0^l \frac{\sigma_1}{\sigma_{eq}}\frac{D}{\sqrt{s_c}u_\tau}dl = C. \tag{7}$$

It has been assumed here that $l = 0$ at the entrance to the plastic zone.

III. PLANE STRAIN DRAWING

A schematic diagram of the plane strain drawing process is shown in Fig. 1. The solution for flow of rigid perfectly/plastic material through a wedge-shaped channel given in [26] can be used to find an approximate solution of this problem. In particular, the stress components in the plane polar coordinate system $r\varphi$ are given by

$$\frac{\sigma_{rr}}{2\tau_s} = -c\ln\frac{r}{r_0} + \frac{1}{2}\cos 2\psi - \frac{1}{2}c\ln\left(c-\cos 2\psi\right) + A,$$
$$\frac{\sigma_{\varphi\varphi}}{2\tau_s} = \frac{\sigma_{rr}}{2\tau_s} - \cos 2\psi, \quad \frac{\sigma_{r\varphi}}{\tau_s} = \sin 2\psi. \tag{8}$$

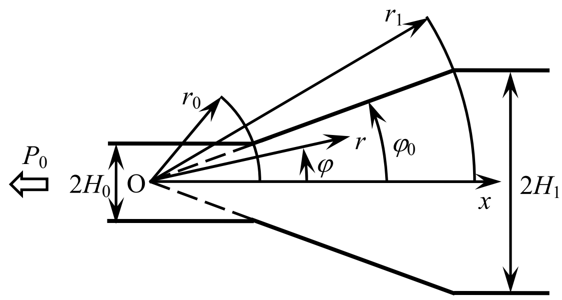

Fig.1 Schematic diagram of the plane strain drawing process.

where ψ is the angle between a radius and the direction of σ_1 measured from the radius anti-clockwise, $c > 0$ and A are constants of integration, r_1 is the value of r at the entrance to the die, and r_0 is the value of r at the exit from the die. By assumption, ψ depends only on φ and this dependence is determined by the following equation

$$d\psi/d\varphi = c\sec 2\psi - 1. \tag{9}$$

If the friction law (3) applies then ψ ranges between 0 and $\pi/4$ in the interval $0 \le \varphi \le \varphi_0$ where $2\varphi_0$ is the total angle of the die (Fig. 1). The circumferential velocity vanishes and the radial velocity is given by

$$u_r = -\frac{B}{r\left(c-\cos 2\psi\right)}, \tag{10}$$

where B is a positive constant. Using equations (9) and (10) the equivalent strain rate can be written in the form

$$\xi_{eq} = \frac{2}{\sqrt{3}}\frac{B}{r^2\left(c-\cos 2\psi\right)\cos 2\psi}. \tag{11}$$

It follows from equations (3) and (8) that the maximum friction law in the case under consideration becomes

$$\psi = \pi/4 \tag{12}$$

at $\varphi = \varphi_0$. Substituting equation (12) into equation (11) shows that $\xi_{eq} \to \infty$ as $\varphi \to \varphi_0$. Moreover, expanding the denominator of equation (11) in a series in the vicinity of the point $\psi = \pi/4$ gives

$$\xi_{eq} = \frac{B}{\sqrt{3}cr^2}\left(\frac{\pi}{4}-\psi\right)^{-1} + o\left[\left(\frac{\pi}{4}-\psi\right)^{-1}\right], \qquad \psi \to \frac{\pi}{4}. \tag{13}$$

On the other hand, integrating equation (9) and using the boundary condition (12) lead to

$$\pi/4 - \psi = \sqrt{c}\left(\varphi_0-\varphi\right)^{1/2} + o\left(\sqrt{\varphi_0-\varphi}\right), \qquad \varphi \to \varphi_0. \tag{14}$$

Combining equations (13) and (14) results in

$$\xi_{eq} = \frac{B(\theta_0-\theta)^{-1/2}}{\sqrt{3}c^{3/2}r^2} + o\left[(\theta_0-\theta)^{-1/2}\right], \qquad \theta\to\theta_0 . \tag{15}$$

Comparing the general representation of the equivalent strain rate in the vicinity of maximum friction surfaces in the form of equation (2) and equation (15) gives

$$D = \frac{B}{\sqrt{3}c^{3/2}r^{3/2}} . \tag{16}$$

There are two boundary conditions to find a constant of integration involved in the solution of equation (9) and the value of c. One of this conditions is given by equation (12) and the other is the symmetry condition in the form $\psi = 0$ for $\varphi = 0$. Using these conditions and equation (9) one gets

$$\varphi_0 = \int_0^{\pi/4} \frac{\cos 2\psi}{(c-\cos 2\psi)} d\psi \tag{17}$$

This equation should be solved for c numerically. Assume that the magnitude of the material flux per unit length is Q. Then, it follows from equation (10) that

$$Q = B\int_0^{\varphi_0} \frac{d\varphi}{(c-\cos 2\psi)} . \tag{18}$$

Using equation (9) integration with respect to φ in equation (18) can be replaced with integration with respect to ψ. Then,

$$B = Q\left[\int_0^{\pi/4} \frac{\cos 2\psi}{(c-\cos 2\psi)^2} d\psi\right]^{-1} . \tag{19}$$

Excluding c and B in equation (16) by means of equation (19) and the solution to equation (17) it is possible to find the dependence of the strain rate intensity factor on Q and φ_0. It is clear that the dependence on Q is linear. Therefore, it is convenient to introduce the dimensionless strain rate intensity factor in the form

$$d = \frac{Dr^{3/2}}{Q} = \frac{1}{\sqrt{3}c^{3/2}}\left[\int_0^{\pi/4} \frac{\cos 2\psi}{(c-\cos 2\psi)^2} d\psi\right]^{-1} . \tag{20}$$

The dependence of d on φ_0 is illustrated in Fig. 2. The value of A involved in equation (8) is determined from the condition that the total horizontal force at the entrance to the die vanishes. This condition has the following form

$$\int_0^{\varphi_0} \left(\sigma_{rr}\cos\varphi - \sigma_{r\varphi}\sin\varphi\right)\Big|_{r=r_1} d\varphi = 0 , \tag{21}$$

Using equation (9) integration with respect to φ in this equation can be replaced with integration with respect to ψ. Also, the stress components involved in equation (21) can be excluded by means of equation (8). Then,

$$A = c\ln\frac{r_1}{r_0} - \frac{1}{2\sin\varphi_0}\int_0^{\pi/4}\left[\begin{matrix}\cos(2\psi+\varphi) - \\ -c\ln(c-\cos 2\psi)\cos\varphi\end{matrix}\right]\frac{\cos 2\psi}{(c-\cos 2\psi)} d\psi . \tag{22}$$

The value of φ in this equation should be excluded by means of the solution to equation (9). After that integrating in equation (22) can be completed numerically.

Using the transformation equations for stress components, the major principal stress at $\varphi = \varphi_0$ (or $\psi = \pi/4$) is determined in the form

$$\sigma_1 = \sigma_{rr} + \sigma_{r\varphi} . \tag{23}$$

It has been taken into account here that $\sigma_{\varphi\varphi} = \sigma_{rr}$ at $\psi = \pi/4$, as follows from equation (8). Substituting equation (8) at $\psi = \pi/4$ into equation (23) gives

$$\frac{\sigma_1}{2\tau_s} = A + \frac{1}{2}(1 - c\ln c) - c\ln\frac{r}{r_0} . \tag{24}$$

The value of u_τ involved in equation (7) is determined from equation (10) with the use of equation (19) as

$$u_\tau = \frac{Q}{rc}\left[\int_0^{\pi/4} \frac{\cos 2\psi}{(c-\cos 2\psi)^2} d\psi\right]^{-1} . \tag{25}$$

Also, $dl = -dr$ in the problem under consideration. Therefore, substituting equations (20), (24) and (25) into equation (7) gives

$$\frac{2}{3\sqrt{c}}\int_r^{r_1}\left[2A + (1-c\ln c) - 2c\ln\frac{r}{r_0}\right]\frac{1}{\sqrt{r}\sqrt{s_c}} dr = C . \tag{26}$$

It has been assumed here that the material obeys the Mises yield criterion and, therefore, $\sigma_{eq} = \sqrt{3}\tau_s$.

Several general equations for determining the value of s_c have been proposed in [25]. It is important to mention that the integral involved in equation (26) is improper and its convergence is an additional requirement imposed on any equation for s_c compatible with the fracture criterion proposed. The simplest equation for s_c given in [25] does not satisfy this requirement. Another equation for s_c proposed in this work is

$$ds_c/dt = D\sqrt{s_m}\,\Phi(s_c/s_m) , \tag{27}$$

where s_m is a material constant and Φ is a material function of its argument. In the case of stationary processes equation (27) takes the following form

$$\frac{ds_c}{dl} = \frac{D\sqrt{s_m}}{u_\tau}\Phi\left(\frac{s_c}{s_m}\right) . \tag{28}$$

At the present time, no experimental data are available to determine s_m and Φ. The physical meaning of s_m is that it is the maximum possible thickness of the layer of intensive plastic deformation. Therefore, the simplest function $\Phi\left(s_c/s_m\right)$ satisfying all the necessary requirements is

$$\Phi\left(s_c/s_{\mathrm{m}}\right)=\alpha\left(1-s_c/s_m\right), \qquad (29)$$

where α is a material constant. Substituting equation (29) into equation (28) gives

$$\frac{ds_c}{dl}=\frac{\alpha D\sqrt{s_m}}{u_\tau}\left(1-\frac{s_c}{s_m}\right). \qquad (30)$$

Excluding the strain rate intensity factor in this equation by means of equation (20) and u_τ by means of equation (25) results, with the use of equation (19), in

$$\frac{ds_c}{dr}=-\frac{\alpha\sqrt{s_m}}{\sqrt{3cr}}\left(1-\frac{s_c}{s_m}\right). \qquad (31)$$

at $\psi=\pi/4$. Integrating this equation with the use of the boundary condition $s_c=0$ at $r=r_1$ leads to

$$\frac{s_c}{s_m}=1-\exp\left[-\frac{2\alpha}{\sqrt{3c}}\left(\sqrt{\frac{r_1}{s_m}}-\sqrt{\frac{r}{s_m}}\right)\right]. \qquad (32)$$

Substituting equation (32) into equation (26) gives

$$\frac{2}{3\sqrt{c}}J=\frac{C}{\sqrt{\beta}},$$

$$J=\int_q^1\frac{2A+\left(1-c\ln c\right)-2c\ln\left(\dfrac{\rho}{q}\right)}{\sqrt{\rho}\sqrt{1-\exp\left[-\dfrac{2\alpha\sqrt{\beta}}{\sqrt{3c}}\left(1-\sqrt{\rho}\right)\right]}}d\rho \qquad (33)$$

at the exit from the die. Here $q=r_0/r_1$ and $\beta=r_1/s_m$. It is obvious that $q=H_0/H_1$ (Fig. 1). The integral in equation (33) is improper but it is easy to show convergence. The variation of J with φ_0 is depicted in Fig.3. Note that β is a very large number since s_m is very small. Therefore, the right hand side of equation (33) is very small. On the other hand, it is seen from Fig.3 that the value of J in general differs from zero significantly. Therefore, due to a lack of experimental data, it is possible to approximately estimate the instant of fraction initiation by means of the condition $J=0$. Thus the intersection of the curves with the horizontal axis determines the die angles at which fracture initiates near the friction surface at a given value of reduction (Fig. 3). It is seen from this figure that the range of safe angles increases with the reduction. It is however necessary to mention that the complete analysis of the process requires consideration of other fracture modes such as, for example, the formation of central bursting [27].

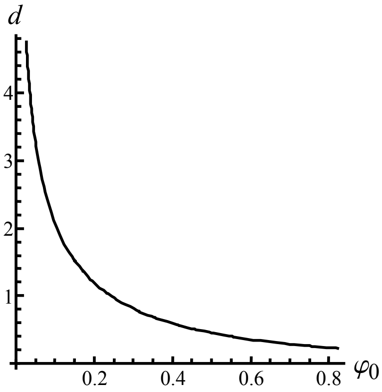

Fig.2 Variation of the dimensionless strain rate intensity factor with die angle.

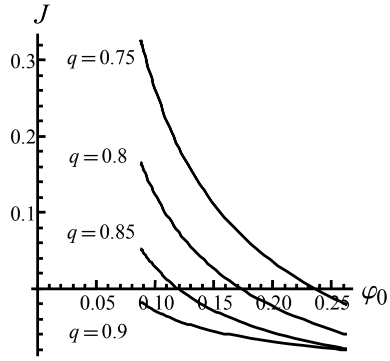

Fig.3 Effect of die angle and reduction on the fracture initiation according to the new fracture criterion.

III. DISCUSSION AND CONCLUSIONS

A new non-local ductile fracture criterion has been proposed. The criterion allows one to overcome the difficulty related to the singular behaviour of the equivalent strain rate near maximum friction surfaces. An example to illustrate the procedure to apply the new criterion has been given. For application to real metal forming processes, it is necessary to propose and carry out a special experimental program to determine the material constants involved in the model. It is obvious that conventional tests are not appropriate for this purpose since the model includes the strain rate intensity factor which is associated with infinite equivalent strain rate at maximum friction surfaces.

ACKNOWLEDGMENT

The research described in this paper was supported by grants RFBR-09-01-92005 and GK11.519.11.3015 (Russia). A part of this work was done while the second author was with National Chung Cheng University (Taiwan) as a research scholar under the recruitment program supported by the National Science Council of Taiwan (contact 99-2811-E-194-009).

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