Interpolation-Integrated Contouring Control of Biaxial Systems

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Abstract - This study is concerned with the contouring control of biaxial motion systems. It is well known that contour errors depend not only on the controller, but also on the reference command. Two problems are investigated in this paper. First, it is to find the reference command that will yield the minimum contour error. This is the problem of interpolation and acceleration/deceleration. Second, given a reference command, it is to find a modified reference command so that the system output will be as close to the reference command as possible. The design of the modified reference command utilizes the inverse model of the closed-loop system. In other words, the modified reference command can be considered as the output of the inverse system with the desired reference command as the input. It can be obtained by the convolution technique.

Keywords - contouring control, interpolation, biaxial systems

I. INTRODUCTION

Biaxial systems are widely used in industrial applications. They are commonly seen in CNC machine tools. To achieve high speed and high accuracy due to high precision and high productivity requirement nowadays, motion control of biaxial systems is a highlighted issue since last two decades. Motion control of CNC machine could be divided in two parts: 1) path planning; 2) servo loop control. Given a desired contour, the path planning is to generate appropriate command signals for each axis, includes interpolation and acceleration/deceleration planning. On the other hand, the objective of servo loop control is to make the output position of each axis follow the command signal. Contour error may arise in both stages of path planning and servo-loop control. This study is to address the issue of reducing contour errors by proper path planning and servo-loop control.

Some people tried to reduce the tracking error of single axis to reduce the contouring error, so the feedback controller has been designed. The most popular controller in industry is PID (proportional & integral & differential) controller [5] which has been invented by Minorsky in the early twentieth century. However, it is difficult to tune the optimal control parameters. Kaan Erkorkmaz [2] [3][4] used Kalman Filter to be the observer to obtain position, velocity and disturbance estimate and a pole placement controller to be the feedback controller. Finally, adding a zero phase error tracking controller (ZPETC) to widen the bandwidth and make the performance of contouring better, but the major difficulty is the computational complexity of designing the controllers and observer.

For multi-axis system what we care is the contour error and the contour error does not proportional to tracking error. To reduce the contour error the objective does not focus on reducing tracking error. Ho [1] decomposed the error from x-y directions to normal and tangential direction. The tangential direction is related to move the tool along the trajectory at the desired feedrate and the normal direction is related to maintain the position of target trajectory. In order to give an accurate and simple form for contour error model, Wu [6] developed equivalent errors model. It was used to design feedback stabilization controller (FSC), sliding mode controller (SMC) and integral sliding mode controller (ISMC) to reduce the contouring error of multi-axis system. The equivalent error model could handle the contour error problem directly and more convenient for computation. Further, it is both suitable for linear system and non-linear system. The major problem for equivalent error model is that it is required to have the path equation first.

Further, an interpolator will affect the error so that how to design a good interpolator had become an important study for contouring. Conventional CNC machines only provide linear and circular interpolators were segment a curve into many small parts and linearized segments are sent to CNC system. Such process would cause some problems, such as discontinuity for segmentation, motion speed becomes uneven.

Conventionally, interpolator and controller are designed separately. However, there are coupling effects between interpolator and controller. In other words, given a closed-loop system, the interpolator should generate the command signals that match the dynamics of the closed-loop system. The objective of this study is to design the reference commands by considering the dynamics of the closed-loop system. In this research, we only discuss about the case of tracking a linear path and we will design the reference command by using three popular acceleration/deceleration scheme. Finally, we will use these reference commands to design our modified reference commands that make the system output equal to reference command and this is the objective of this research——to find out the optimal command that make the contour error minimum by regarding the system dynamics.

II. DESIGN OF REFERENCE COMMAND

All the smooth parameter curves could be regarded as the acceleration/deceleration schemes. In our study, we will introduce three popular acceleration/deceleration schemes (i.e. linear type, exponential type and bell-shaped type) and use these schemes to design the reference commands of linear path. The bases of comparison about three acceleration/deceleration schemes are

—the same average velocity

—the same maximum feedrate (15 mm/sec).

—the same acceleration time (1/3 sec)

The case is to track a linear path from 0 to 10 mm in one sec.

The reference command of bell-shaped type is

$$r_{bel}(t)=\begin{cases} 120t^3 & (0<t\le \frac{1}{12}) \\ 2.5(t-\frac{1}{12})+30(t-\frac{1}{12})^2 & \\ \quad +0.07 & (\frac{1}{12}<t\le\frac{1}{4}) \\ 12.5(t-\frac{1}{4})+30(t-\frac{1}{4})^2 & \\ \quad -120(t-\frac{1}{4})^3+1.32 & (\frac{1}{4}<t\le\frac{1}{3}) \\ 15(t-\frac{1}{3})+2.5 & (\frac{1}{3}<t\le\frac{2}{3}) \\ 15(t-\frac{2}{3})-120(t-\frac{2}{3})^3 & \\ \quad +7.5 & (\frac{2}{3}<t\le\frac{3}{4}) \\ 12.5(t-\frac{3}{4})-30(t-\frac{3}{4})^2 & \\ \quad +8.68 & (\frac{3}{4}<t\le\frac{11}{12}) \\ 120(t-\frac{11}{12})^3-30(t-\frac{11}{12})^2 & \\ \quad +2.5(t-\frac{11}{12})+9.93 & (\frac{11}{12}<t\le 1) \end{cases} \tag{1}$$

The reference command of linear type is

$$r_{lr}(t)=\begin{cases} 22.5t^2 & (0<t\le\frac{1}{3}) \\ 15(t-\frac{1}{3})+2.5 & (\frac{1}{3}<t\le\frac{2}{3}) \\ -22.5(t-\frac{2}{3})^2+15(t-\frac{2}{3})+7.5 & (\frac{2}{3}<t\le 1) \end{cases} \tag{2}$$

The reference command of exponential type is

$$r_{\exp}(t)=\begin{cases} 15t+\frac{5}{4}(e^{-12t}-1) & (0<t\le\frac{1}{3}) \\ 15(t-\frac{1}{3})+3.75 & (\frac{1}{3}<t\le\frac{2}{3}) \\ \frac{5}{4}(1-e^{-12(t-\frac{2}{3})})+8.75 & (\frac{2}{3}<t\le 1) \end{cases} \tag{3}$$

III. DESIGN OF MODIFIED REFERENCE COMMAND

Since we have three reference commands, we will use them to design the modified reference commands.

The idea of modified reference command is to design a reference input $u(t)$ that make the output $y(t)$ equal to our reference command $x(t)$ by placing an inverse of plant forward, Fig. 1.

First, we take a polynomial left comprise fraction

$$G(s)=\frac{N(s)}{D(s)} \tag{4}$$

where

$$N(s)=b_m s^m+b_{m-1}s^{m-1}+\cdots+b_1 s+b_0$$

$$D(s)=s^n+a_{n-1}s^{n-1}+\cdots+a_1 s+a_0$$

Our idea is to give a reference command $x(t)$ pass through the inverse system of plant and the output $u(t)$ will be our modified reference command.

The S-domain of modified reference command is

$$U(s)=G^{-1}(s)X(s)=\frac{D(s)}{N(s)}X(s) \tag{5}$$

Since the reference command $x(t)$ and the closed-loop system $G(s)$ are known, we explain (5) to be a differential equation

$$\begin{aligned}&\left(b_m u^{(m)}(t)+b_{m-1}u^{(m-1)}(t)+\cdots+b_1\dot{u}(t)+b_0\right)\\ &=\left(x^{(n)}(t)+a_{n-1}x^{(n-1)}(t)+\cdots+a_1\dot{x}(t)+a_0\right)\end{aligned} \tag{6}$$

Since the reference command $x(t)$ and parameters $a_i\ (i = 0,1,2........n)$ are known, the right hand side of equation (6) can be calculated off-line and equation (6) will become an nth order differential equation.

Here we let

$$\hat{x}(t) = x^{(n)}(t) + a_{n-1}x^{(n-1)}(t) + \cdots + a_1\dot{x}(t) + a_0 \quad (7)$$

We could use the convolution technique and get the modified reference command as

$$u(t) = h(t)\hat{x}(0) + \int_0^t h(t-\lambda)\hat{x}(\lambda)\, d\lambda \quad (8)$$

where $h(t)$ is the impulse response of $\frac{1}{N(s)}$.

IV. EXAMPLE SIMULATION

We will use the method of convolution integral to design the reference input and given some systems to do simulation to check whether our approach work well or not. Here we will use first order and second order systems to do some simulations based on convolution integral method. The software we use is SIMULINK of MATLAB. The modified reference commands will design from reference commands equation (1), (2) and (3). Our goal is to track a path from 0 to 10 mm in one sec.

The Root-Mean-Square Error

$$\mathrm{RMS}[\varepsilon] = \left(\int_0^1 \varepsilon^2(t)\, dt \right)^{\frac{1}{2}} \quad (9)$$

is a index used as a numerical measure of tracking performance. The systems we suppose in this section are random.

A. minimum phase system

First, we discuss about minimum phase systems whose relative degree is zero and greater than one.

Given a first order system $\frac{s+8}{s+3}$ whose relative degree is zero and a second order system $\frac{s+1}{s^2+2s+3}$ whose relative degree is one, the modified reference commands could get by using convolution technique.

The results of RMS error are listed in Table 1.

B. non-minimum phase system

For non-minimum phase system we also give some systems whose relative degree is zero and one.

These systems are $\frac{s-8}{s+3}$, $\frac{s-2}{s+5}$, $\frac{s-1}{s^2+2s+3}$, $\frac{s-2}{s^2+3s+8}$ and $\frac{s-5}{s^2+3s+8}$. The results of RMS error are listed in Table 2.

According to the results, we find that the tracking errors of three modified reference commands are about $10^{-6}\,m$ for minimum phase system. We could get that the convolution technique is work for minimum phase system to design the modified reference command based on the transfer function of whole closed-loop system.

According to the result listed in Table 2, we could know the values of numerator of the system whose relative degree is greater than one have less effect on tracking errors. So the convolution technique still works for non-minimum phase systems whose relative degree is one. For non-minimum phase system whose relative degree is zero it still has some problem to overcome for implementation.

V. EXPERIMENTAL RESULT

A. Experimental Setup

The overall experimental setup is sketched in Fig.2 is the corresponding photo DS1103 PPC controller board mounted on a PC (together with software Matlab/Simulink and Control Desk) is responsible for generating command input, receiving position feedback signal from linear encoder, and calculating the control input to be sent into each axial servo drive. Each axial drive will transmit the power to drive the motor and close the internal feedback loop, which depends on the control mode selected, i.e. position, velocity and torque control mode. The motors mounted on base provide the torque to drive the ballscrews, which cause the XY table to move, since ballscrews are connected directly to XY table.

B. Results and Disscussion

For each modified reference command, 10 runs of experimental data will be collected, and the average RMS error will be the index for tracking performance. The figures we show are the best one of the experimental results of each kind of three modified reference commands. The tracking result with modified commands $u_{lr}(t)$, $u_{\exp}(t)$, and $u_{bell}(t)$ are shown in Fig.3, Fig.4, and Fig.5, respectively. The RMS errors of tracking error of per modified command are listed in Table 4. These results clearly verify the theoretical analysis.

VI. CONCLUSIONS

Given a biaxial system and desired path, both controller and reference command can affect the contouring accuracy. It is assumed that the controller has been designed. In other words, we are given a closed-loop system. In the CNC configuration, given a desired path, the reference command is obtained by interpolator incorporated with a proper acceleration/deceleration scheme. Three popular acceleration/deceleration schemes have been examined i.e. linear type, exponential type and bell-shaped type. They are compared under the conditions that they will result in the same average feedrate and the same maximum feedrate. Numerical results for a first order system showed that the bell-shaped type is better than the other two types.

Also, a method to design the modified reference command has been proposed in this thesis. The modified reference command is designed so that the tracking error of the system output and the desired reference command is minimum. The method is similar to the method of feedforward controller. In other words, it made use of the inverse system of the regarded as the output of the inverse system due to the desired reference command. Therefore, it can be obtained easily by the convolution of the reference command with the impulse response function of the inverse system.

The difference between the feedforward controller and the modified reference command is that the latter is obtained off-line and is then take as the reference input to the closed-loop system. This method has been verified numerically both for minimum phase and non-minimum phase systems. It works well for minimum phase system. However, for non-minimum phase systems, it is sensitive to noise and numerical errors. This method has also been verified experimentally on a biaxial system.

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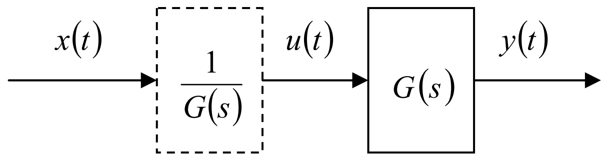

Fig.1 Block diagram of modified reference command

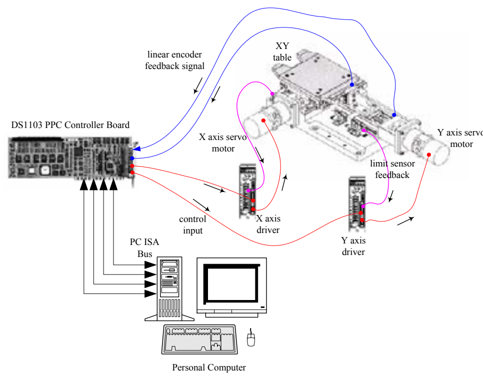

Fig.2 Experimental setup for contouring control of biaxial system

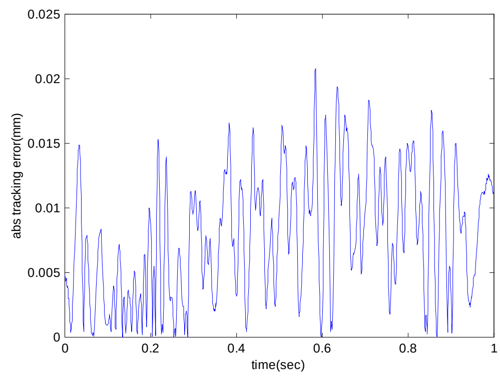

Fig.3 The tracking error of $u_{lr}(t)$ about x-axis

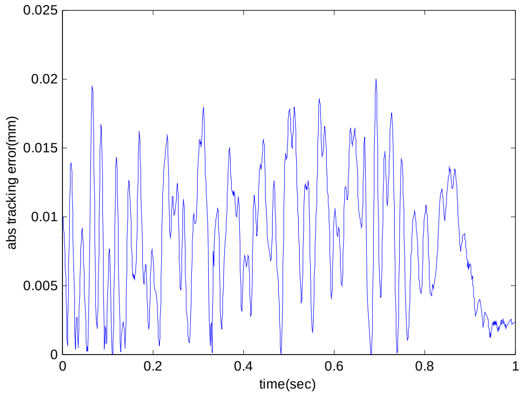

Fig.4 The tracking error of $u_{\exp}(t)$ about x-axis

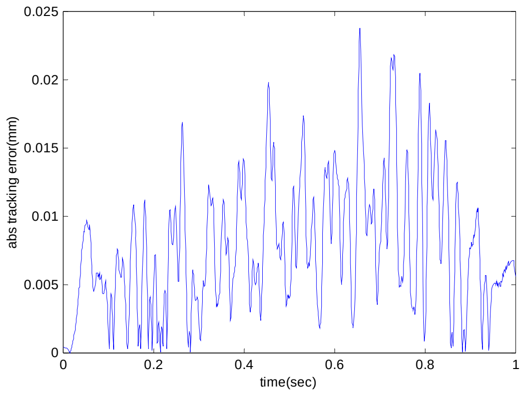

Fig.5 The tracking error of $u_{bell}(t)$ about x-axis

TABLE 1 RMS errors with various reference inputs

system	Linear type	Exponential type	Bell-shape type
$\frac{s+3}{s+8}$	$3.25\times10^{-6}m$	$2.34\times10^{-6}m$	$3.17\times10^{-6}m$
$\frac{s+1}{s^2+2s+3}$	$5.59\times10^{-6}m$	$4.45\times10^{-6}m$	$5.72\times10^{-6}m$

TABLE 2 RMS error of non-minimum phase system

System	Linear type	Exponential type	Bell-shape type
$\frac{s-8}{s+3}$	$2.68\times10^{-3}m$	$4.68\times10^{-3}m$	$2.42\times10^{-3}m$
$\frac{s-2}{s+5}$	$4.67\times10^{-5}m$	$5.79\times10^{-5}m$	$4.66\times10^{-5}m$
$\frac{s-1}{s^2+2s+3}$	$5.59\times10^{-6}m$	$4.48\times10^{-6}m$	$5.71\times10^{-6}m$
$\frac{s-2}{s^2+3s+8}$	$5.60\times10^{-6}m$	$4.51\times10^{-6}m$	$5.66\times10^{-6}m$
$\frac{s-5}{s^2+3s+8}$	$5.69\times10^{-6}m$	$4.55\times10^{-6}m$	$5.74\times10^{-6}m$

TABLE 3 The RMS errors of experimental results

	Linear type	Exponential type	Bell-shape type
1	$1.54\times10^{-5}m$	$1.03\times10^{-5}m$	$1.09\times10^{-5}m$
2	$1.63\times10^{-5}m$	$1.08\times10^{-5}m$	$1.26\times10^{-5}m$
3	$1.46\times10^{-5}m$	$1.34\times10^{-5}m$	$0.99\times10^{-5}m$
4	$1.00\times10^{-5}m$	$0.96\times10^{-5}m$	$1.76\times10^{-5}m$
5	$1.20\times10^{-5}m$	$1.16\times10^{-5}m$	$0.93\times10^{-5}m$
6	$1.00\times10^{-5}m$	$1.05\times10^{-5}m$	$0.91\times10^{-5}m$
7	$1.14\times10^{-5}m$	$1.54\times10^{-5}m$	$0.94\times10^{-5}m$
8	$1.66\times10^{-5}m$	$1.11\times10^{-5}m$	$0.95\times10^{-5}m$
9	$1.02\times10^{-5}m$	$1.07\times10^{-5}m$	$1.00\times10^{-5}m$
10	$0.92\times10^{-5}m$	$1.52\times10^{-5}m$	$0.99\times10^{-5}m$
Avg	$1.26\times10^{-5}m$	$1.19\times10^{-5}m$	$1.08\times10^{-5}m$