A Change of Curved Direction of Distortional Strand to An Opposite Direction in Stereolithography

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Abstract - Products in stereolithograph are produced by many strands, which are curved strings of cured resin by an ultraviolet ray or a visible ray, and the strands are laminated to 3 dimensional products. A single strand of cured resin is curved to an exposed side of the strand. However it was found that there existed a case where curved direction was changed to opposite side on an exposed surface of strand. A finite element code has been developed to analyze this phenomenon. When liquid resin is cured by exposure of ray, a width of which is wide and a penetrating depth of which is rather shallow, there exists much volume of incompletely cured region at opposite side of strand. The incomplete region has an ability to shrinkage further, which makes the strand curve oppositely after along lapse of time.

Keywords - Stereolithography, strand, FEM, distortion, resin

I. INTRODUCTION

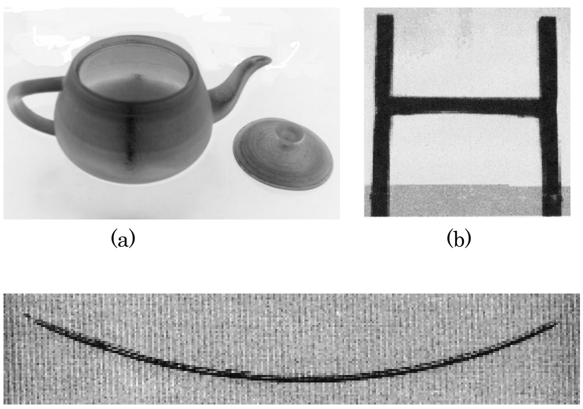
Three dimensional products in stereolithograph (SL) [1] shown in Figs. 1(a) and (b) are produced by cured resin due to an exposure of ultraviolet or visible ray, and by scanning of their laser beams on a surface of liquid resin. A lot of scanning of their laser beams makes plane layer on a surface of liquid and additional scanning makes new plane layer on the former layer of cured resin. From this lamination of scanning, 3 dimensional products of solid are possible to be produced directly from 3 dimensional computer data of CAD and MRI. When liquid resin is cured, shrinkage occurs, which produces distortion of products and generates serious problems in precision of products.

Products in stereolithograph are produced by a strand like a curved string of cured resin as shown in Fig. 1(c), and a lot of strand is laminated to 3 dimensional products. A single strand is curved to an exposed side of the strand generally. However it is found by one of author in an experiment 15 years ago [2] that curved direction was changed to opposite side. A code of finite element method (FEM) has been developed by the authors [3-5] to analyze a distortion of a single strand and of 3 dimensional products made of strand.

It is found from numerical analysis by using FEM that curved direction of a single strand is changed to opposite situation [6]. When liquid resin is cured by exposure of ray, a width of which is wide and a penetrating depth of which is rather shallow, there exists much volume of incomplete cured region at opposite side of strand, which is rather soft like gelling material. The incomplete region has an ability to shrinkage further which makes the strand curve oppositely after a long lapse of time.

II. DEVELOPMENT OF FEM CODE

Curl distortion of an object produced by stereo-lithography occurs due to shrinkage of cured resin. A mechanism of curing process of resin was studied [7,8] using a finite element method [9]. The authors [3-6] also developed a FEM code to study a mechanism of the curl distortion, solidification of liquid resin, and curing process of resin. In this report, a gradual progression of curing due to an irradiation of sunlight has been developed for solving a direction of curl distortion in addition to a development of eight terms mentioned in previous paper [4].



(a) (b) (c)

Fig. 1 3-dimensional models (a), (b) produced by SL process and a single strand (c) created by exposure of ray.

A. Elastic and Visco -Plastic Material

Curing process of resin is solved by a finite element code. Total strain $\{\varepsilon\}$ of an elastic and visco-plastic material is composed of elastic strain $\{\varepsilon^e\}$, visco-plastic strain $\{\varepsilon^v\}$, strain of thermal expansion $\{\varepsilon^T\}$, and shrinkage strain $\{\varepsilon^r\}$ due to irradiation, and is expressed as

$$\{\Delta\varepsilon\}=\{\Delta\varepsilon^e\}+\{\Delta\varepsilon^v\}+\{\Delta\varepsilon^T\}+\{\Delta\varepsilon^r\} \quad (1)$$

where Δ denotes an incremental amount of variable. Stress-strain matrix and strain-displacement matrix are expressed by $[D]$ and $[B]$, respectively. Using a coefficient of thermal expansion a, temperature T, exposure of ray E, and a coefficient of bulk modulus K_c, stiffness matrix $[K]$ and effective nodal force $\{\Delta f\}$ are given as follow [9].

$$[K]=\int_v dv\{[B]^T[D][B]\} \quad (2)$$

$$\{\Delta f\}=\int_v dv[[B]^T\{\Delta\sigma\}+[B]^T 3K_c(a\{\Delta T\}+\{\Delta\varepsilon^r(E)\}) +[B]^T[D]\{\Delta\varepsilon^v\}] \quad (3)$$

An equilibrium equation of force is expressed as

$$[K]\{\Delta U\}-\{\Delta f\}=0. \quad (4)$$

B. Constitutive Equation of Cured Resin

Liquid resin cured by an exposure of ray is solidified with heat generation. Shrinkage of the cured resin occurs rapidly for a short elapsed time of a few second after exposure. But thermal shrinkage occurs slowly for a long elapsed time of 20 seconds and other shrinkage due to an irradiation of sunlight is assumed to occur gradually for a very long elapsed time of weeks or months after exposure.

On a curing process of resin, shrinkage strain ε_o and elastic modulus Y due to solidification of polymer are written [1] by an amount of exposure E as, respectively,

$$\varepsilon_o=\varepsilon_{oi}+(\varepsilon_f-\varepsilon_{oi})[1-\exp(-\alpha t)] \quad (5)$$

$$\varepsilon_f=\varepsilon_{om}[1-\exp\{-\beta(E/E_c-1)^p\}\exp\{-(\gamma t)^q\}] \quad (6)$$

$$Y=Y_i+(Y_f-Y_i)[1-\exp(-\alpha t)] \quad (7)$$

$$Y_f=(Y_{max}-Y_{min})[1-\exp\{-\beta(E/E_c-1)^p\} \times\exp\{-(\gamma t)^q\}]f_Y(T)+Y_{min}. \quad (8)$$

ε_f and Y_f are a final cure shrinkage and a final Young's modulus under an exposure E as shown in Fig. 2. A suffix i denotes a shrinkage strain and an elastic constant at i th step. t is a time. α and β is a coefficient of the cure rate concerning a time and a exposure energy E of ray, respectively. ε_{om} is a maximum cure shrinkage and Y_{MAX} is a maximum elastic modulus. E_c is a critical threshold exposure below which curing of liquid resin does not occur, that is, $\varepsilon_f=0$ and $Y_f=0$ when $E<E_c$. An elastic modulus Y, which decreases drastically during glass transition temperature, thermal expansion and Poisson ratio are written in previous paper [4].

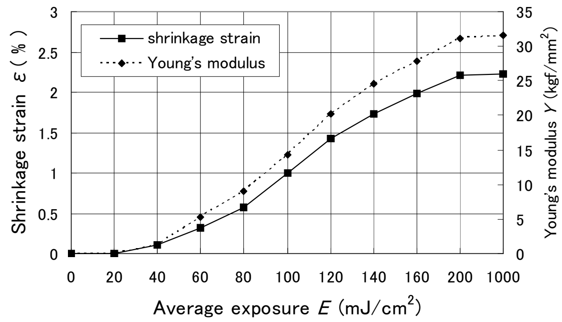


Fig. 2 Relationship of cure shrinkage and Young's modulus against laser exposure.

C. Rate of Shrinkage in Week and Month

Distortion of cured resin during a long elapsed time is investigated in this report. Shrinkage after exposure is assumed to occur gradually for a very long elapsed time of weeks, months or years due to an irradiation of sunlight outside. The authors consider that a constitutive equation may be proper to include a mechanism of the shrinkage in order to explain a phenomenon of curl direction of strand. Fig. 3 shows curves of shrinkage strain ε against time t obtained from (5) and (6) at constant exposure energy. The term $\exp\{-(\gamma t)^q\}$ of (6) expresses a rate of shrinkage per second, which produces a large distortion at $\gamma=10^{-5}\,sec^{-1}$ when $t>10^5$ sec. Time of 10^5 second is equal to 1.2 day. A shrinkage strain increases to reach a value of ε_f at $\gamma=0$ of Eq. (6) as shown by solid curves in Fig. 3. When γ is not equal to 0, its strain reaches a final shrinkage ε_{om} after a long lapse of weeks, months or years shown by dotted curves in Fig. 3. This term of $\exp\{-(\gamma t)^q\}$ is possible to makes a reversible movement of curved direction of a

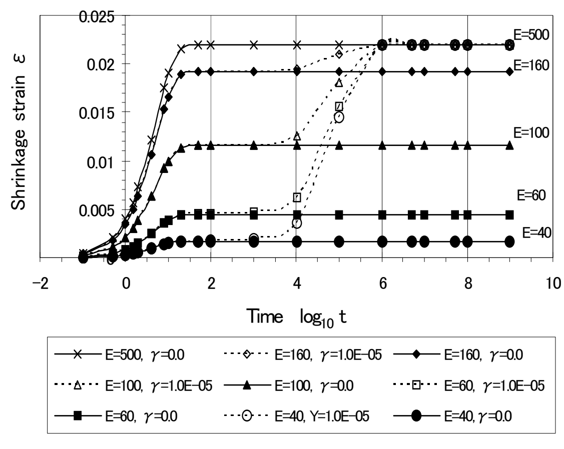


Fig. 3 Shrinkage strain ε - time t curves depending on exposure energy E [mJ/cm^2]. Number of second is 3600s for an hour, 8.64×10^4s for a day, 6.05×10^5s for a week, 2.59×10^6s for a month, 3.15×10^7s for a year, and 3.15×10^8 s for 10 years.

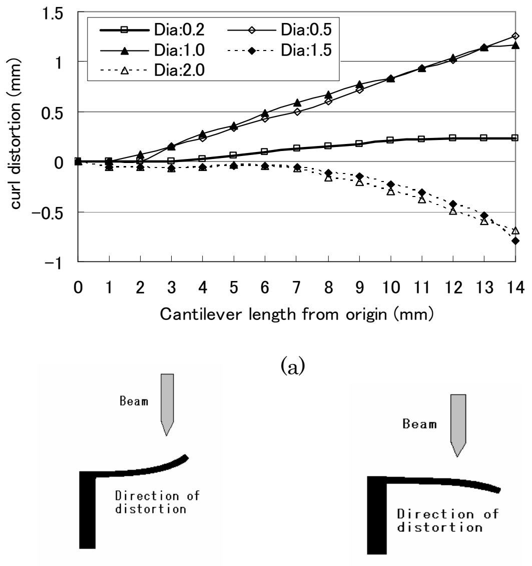
single strand.

III. SINGLE STRAND IN EXPERIMENTS

A. Cantilever Specimen

It was found by one of the authors in experiment 15 years ago [2] that curved direction of strand depended on an intensity of beam, a width of beam, and a penetrating depth of beam into resin. He used an equipment of SOUP [1] called as SLA machine to make a double cantilever specimen. The machine builds parts of an object from the bottom to the top, irradiating u. v. ray over the resin. The platform of the objects is indexed down into the liquid resin bath.

Experimental data of distortion of a cantilever are shown in Fig. 4 and TABLE I [2]. A strand curved as concave on a surface of an irradiated side when a diameter of beam was 0.5mm ϕ and 1.0mm ϕ as shown by solid line in Fig. (a) and by Fig. (b). A width of beam was narrow and a penetrating depth of beam was deep, which meant that an intensity of beam was rather strong. A strand curved slightly or was nearly flat when a diameter of beam was 0.2mm ϕ as shown by thick solid line. A width of beam was very narrow and a penetrating depth of beam was deep, which meant that an intensity of beam was considerably strong. A strand curved as convex on a surface of exposure side when a diameter of beam was 1.5mm ϕ and 2.0mm ϕ as shown by a dotted line in Fig.(a) and by Fig.(c). A width of beam was wide and a penetrating depth of beam was rather shallow, which meant that an intensity of beam was weak.



(a)

(b) (c)

Fig. 4 Distribution of curl distortion of cantilever specimen in experiments. Direction is shown in a case (b) where diameters are 0.5 and 1.0mm, and (c) where diameters are 1.5 and 2.0mm.

TABLE I CURE WIDTH AND DEPTH OF STRAND PRODUCED BY IRRADIATION OF BEAM DIAMETER (unit mm)

Beam diameter	0.2	0.5	1	1.5	2
Cure width	0.26	0.5	0.94	1.33	1.62
Cure depth	1.25	1.05	0.9	0.8	0.75

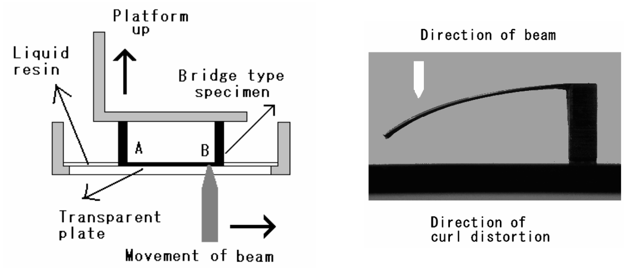
B. Bridge Type Specimen

The authors made a bridge type specimen to confirm the curved direction of strand depending on a width and a depth of irradiating beam. They used an equipment of E−DARTS [10] called as COLAMM system, which is quite different from SLA machine and builds an object from the top to the bottom, irradiating a visible ray from underneath. It forms additional layers from underneath and the platform is indexed up after the layer is completed.

Experimental data of distortion of a cantilever was made from a bridge type specimen shown in Fig. 5(a) [11]. Two piers were produced first by exposure of ray and a part of bridge between the piers was produced next by a scanning of ray. After the bridge specimen was removed from equipments and left a few hour or a day, one of the bridge ends A or B was cut to produce a cantilever specimen as Fig. 5(b). A part of bridge between two piers curved as convex on a surface of an irradiated side, whose direction of curl is same as one of curves of diameter 1.5mm ϕ and 2.0mm ϕ in Fig. 3.

IV. NUMERICAL RESULTS

Two type of mesh division was used for FE analysis, one of which is shown in Fig. 6. Dimension of a whole structure is 10.0×1.0×1.2mm used in a case of neglecting a term γ of shrinkage as γ=0.0 and 10.0×2.0×2.4mm used in a case of considering a term γ of shrinkage as $\gamma=10^{-6}s^{-1}$. Size of a small element is 0.2×0.1×0.1mm and 0.2×0.2×0.2mm, respectively. Total number of elements and nodes are 6000 and 7293 in both cases, respectively.



(a) (b)

Fig. 5 Machine E-DARTS as COLAMM system. Cantilever specimen (b) is set upside down after cutting a corner A or B of bridge type specimen (a).

Exposure energy of beam E is calculated from beer-Lambert law [1] as

$$E(x,0,0)=\{2P_f/(\pi r_0^2 V)\}\exp(-2y^2/r_0^2)\exp(-z/D_p)$$
$$\times \int_{-\infty}^{+\infty}\exp(-2x^2/r_0^2)dx = (2/\pi)^{1/2}P_f/(r_0 V) \qquad (9)$$

where P_f is laser power, r_o is Gaussian half-width of beam, D_p is penetration depth of laser, and V is scanning speed of beam. When P_f is 45mW, V is 10mm/s and r_0 is 0.5mm, E is obtained as 712mJ/cm^2. Constants of constitutive equation (5) to (9) are used for calculation as Ec =10.39 mJ/cm^2, α = 0.2 s^{-1}, ε_{om} = 0.022, Y_{max} = 33.0 kgf/mm^2 at 25℃, Y_{min} = 0.5 kgf/mm^2 at 80℃, ν = 0.3, and D_p is 0.17mm to 0.2mm. Concerning β, γ, p and q, examining values β of them is 0.1, 0.2, 0.01 or 0.018, p is 1 or 2, γ is 0.0 to 10^{-3} s^{-1}, and q is 1 or 0.3.

A. Neglect of Term γ of Shrinkage

Shrinkage due to an irradiation of sunlight is assumed not to occur. A term of γ in (6) and (8) is set as γ = 0.0 sec^{-1} in addition to β = 0.2, p =1 and D_p = 0.17. Numerical results of curl distortion of one-dimensional strand are shown in Figs. 7 and 8 [4]. Fig. 7 shows a deformation of strand of cured resin and Fig. 8 shows distribution of curl distortion along a cantilever length of strand. These results of formation and distortion of cured resin are produced at 30th step of 1.05x10^5 sec after exposure according to an incremental forming. It is shown in Fig. 8 that curl distortion of a strand increases rapidly after exposure and that an amount of increment decreases in near 4 seconds. The calculated results become very close to an experimental result in 26.5 seconds after irradiation. The distribution of curl distortion of calculated results is different from the experimental data slightly as shown by dotted marks in Fig. 8.

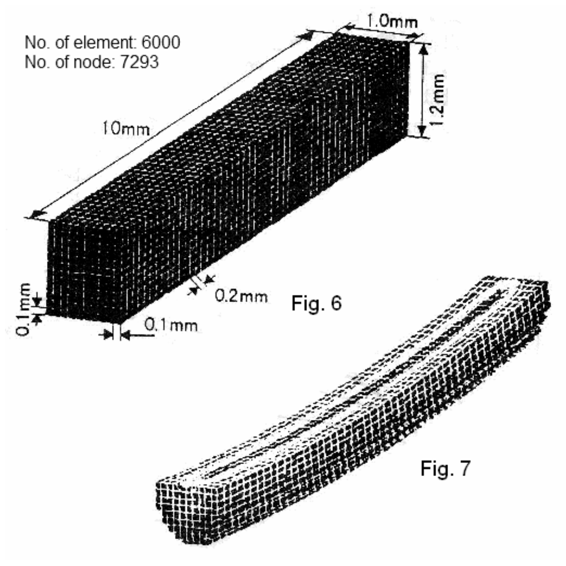


Fig. 6 Mesh division for FE analysis

Fig. 7 Curl distortion of strand

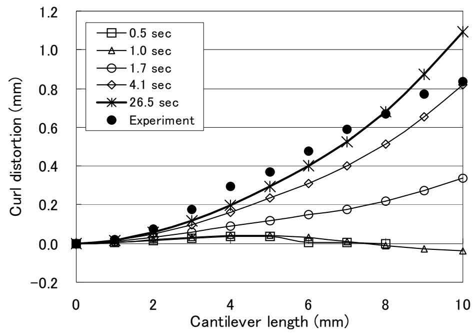


Fig. 8 Numerical result of curl distortion compared with experimental data when γ=0.0 sec^{-1}

B. Consideration of Term γ of Shrinkage

A rate of shrinkage during a long time is γ = 10^{-6} sec^{-1} in addition to β = 0.2, p = 1, q = 1 and D_p = 0.17, which means that time of 10^6 sec is 1.6 weeks. All of cured resin is assumed to reach ε_{om} due to sunlight after a month as shown in Fig. 3. Numerical results of distortion of strand are shown in Figs. 9 and 10, which are simulated by FEM code [6].

Figs. 9(a), (b) and (c) show curl distortions of a strand after 0.5s at 5th step, 1.03x10^{2}s at 20th step, and 10^{7}s at 37th step, respectively. Distributions of curl distortion in Fig. 10 are obtained from these calculated data. A solid line with a mark □ shows formation of cured resin at 5th step after 0.5 sec from starting of irradiation. A half of strand of 10mm is solidified due to cure of resin. A solid line with a mark △ shows distortion of resin at 10th step after 1.0 sec. A whole of strand 10mm is solidified, but curl distortion is very small. A thick solid line with a mark ○ shows

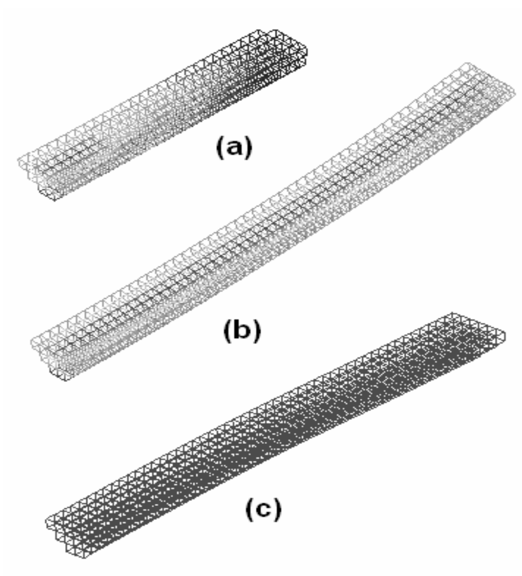


Fig. 9 Deformation and formation of cured resin produced by irradiation of u. v. ray.

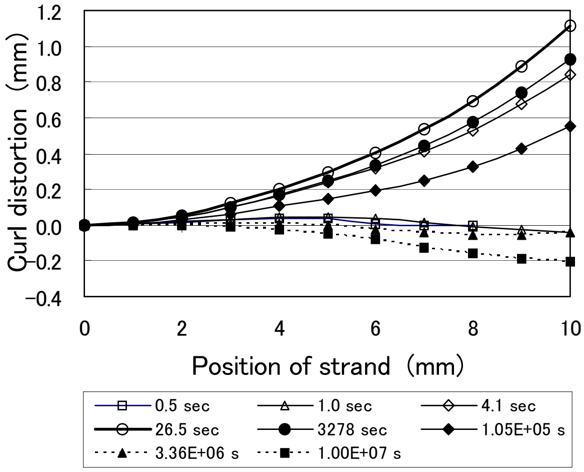


Fig. 10 Curved direction of strand produced by curing. Numerical result shows change of direction of curved strand from concave to convex after a long elapsed time when $\gamma = 10^{-6}$ sec^{-1}.

maximum distortion of strand after 26.5 sec at 18th step. A curved direction of cured resin is concave on an irradiating side. After this time, an amount of distortion decreases, but a curved direction of a line with a mark ◆ after 1.05×10^{5} sec is same concave as the maximum distortional line above-mentioned. After 10^{6} sec of about 1.4 weeks, an amount of distortion decreases gradually. A curved direction of strand is changed from concave to convex as shown in Fig. 10 by a dotted line with a mark ■ on an exposure side.

V. CONCLUSION

3 dimensional products in stereolithograph are made by cured resin due to an exposure of ultraviolet and visible rays, and by scanning of their laser beam on a surface of liquid resin. They are produced by a strand like a curved string of cured resin and a lot of strand is laminated to 3 dimensional products. A single strand is curved to an exposed side of the strand. However it is found by one of author in experiments 15 years ago that curved direction was changed to opposite side. A finite element code (FEM) for solving a distortion of cured resin has been developed to explain this phenomenon. It is necessary to solve formation and distortion of cured resin from a short duration in millisecond and second at exposure to a long duration in day, week, and month after leaving.

It is found from numerical analysis by using FE code that curved direction of a single strand is changed to opposite situation. When liquid resin is cured by exposure of ray, a width of which is wide and a penetrating depth of which is rather shallow, there exists much volume of incomplete cured region at opposite side of strand, which is rather soft like gelling material. The incomplete region has an ability to shrinkage further which makes the strand curve oppositely after a lapse of long time.

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