Exponential Convergence Analysis for Neutral Systems with Constant Delays and Nonlinear Disturbances

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Abstract

In this paper, the problem on delay-dependent exponential convergence for neutral systems with constant delays and nonlinear disturbances is studied. A linear matrix inequality (LMI) based exponential stability criterion is derived by means of the Lyapunov-Krasovskii functional approach and a parameterized model transformation technique. An example is used to illustrate the less conservative result of the proposed approach compared with the previous one.

Keywords： Neutral systems, exponential convergence, nonlinear disturbances, parameterized model transformation.

1. Introduction

It is well known that time delay is commonly encountered in the behavior of many physical processes and very often is the main cause for poor performance and instability of control systems. Therefore, stability problem of time delay systems is a topic of great practical importance that has attracted a considerable amount of interest over the past years. Many methods such as the Lyapunov-Krasovskii functional approach, matrix norm technique, matrix measure technique, Bellman-Gronwall technique, etc., have been proposed in the literature [1-11] for testing the stability of time delay systems. Current efforts on this topic can be divided into two categories, namely delay-dependent stability criteria and delay-independent stability criteria. Generally speaking, the delay-dependent results are often less conservative than the delay-independent results. Moreover, most results are concerning both retarded type delayed systems and neutral-type delayed systems without nonlinear perturbations. To the best of our knowledge, few results have been reported in the literature concerning the problem of delay-dependent robust stability for neutral delayed systems with nonlinear disturbances. In this paper, a delay-dependent criterion for guaranteeing the exponential stability of neutral systems with constant time-delay and nonlinear disturbances is derived by using the Lyapunov-Krasovskii functional method and a parameterized model transformation technique. The stability criterion is formulated in an LMI form. The proposed criterion can be applied to the delay-dependent asymptotic stability testing and is shown to be less conservative than the existing results in the literature [7,8,9,10].

Consider the following neutral time-delay systems with nonlinear disturbances

$$\dot{x}(t)=Ax(t)+Bx(t-h)+C\dot{x}(t-h)+f(x(t),x(t-h),\dot{x}(t-h)) \quad (1)$$

$$x(t)=\phi(t), t\in[-h,\quad 0] \quad (2)$$

where $x(t)\in R^n$ is the state vector; A, B and $C\in R^{n\times n}$ are constant matrices; h is a positive constant time-delay; $\phi(t)$ is a given continuous vector-valued initial function; $f(x(t),x(t-h),\dot{x}(t-h))$ represents the nonlinear disturbances satisfying

$$\|f(x(t),x(t-h),\dot{x}(t-h))\|\le\beta_0\|x(t)\|+\beta_1\|x(t-h)\|+\beta_2\|\dot{x}(t-h)\| \quad (3)$$

where β_0, β_1 and β_2 are positive constants.

Definition 1: The system (1) is said to have a stability degree α (or to be exponentially stable), with $\alpha>0$, if the state of (1) can be written as $x(t)=e^{-\alpha t}z(t)$ and the system governing the state $z(t)$ is asymptotically stable. In this case, the parameter α is called the convergence rate.

To study the exponential stability of the system (1), we introduce a model transformation [1] described by the operator D: $C_0([-h,\quad 0],\quad R^n)\to R^n$ with

$$D(z_t)=z(t)+e^{\alpha h}E\int_{t-h}^{t}z(s)ds-e^{\alpha h}Cz(t-h) \quad (4)$$

where E is a matrix parameter to be chosen.

Remark 1: In view of [2], for the case where $\alpha = 0$, a sufficient condition for the stability of operator $D(z_t)$ is $h\|E\| + \|C\| < 1$. Moreover, an LMI-based sufficient condition is given by [1] as

$$\begin{bmatrix} -I + C^T C & hE^T \\ hE & -I \end{bmatrix} < 0$$

where I is the identity matrix. Besides, in order to analyze the exponential stability of system (1), the following lemma, which significantly reduces the conservatism of the above two conditions, will be used in the derivation of the main result.

Lemma 1[1]: Given a scalar ρ satisfying $0 < \rho < 1$, then for the case where $\alpha = 0$, the operator $D(z_t)$ is stable if there exists a symmetric positive definite matrix X such that the following LMI holds

$$\begin{bmatrix} -\rho X + C^T X C & hE^T X \\ hXE & -X \end{bmatrix} < 0 \qquad (5)$$

2. Main Result

Now, we present a delay-dependent criterion that guarantees the exponential stability of system (1).

Theorem 1: Consider the neutral time-delay system with nonlinear disturbances in (3). Given scalars $\alpha > 0$, ρ satisfying $0 < \rho < 1$, and δ satisfying $0 < \delta < 1$, then for any constant time-delay $h > 0$, this system is exponentially stable with the convergence rate α if there exist symmetric positive definite matrices P, Q_1, Q_2, W, X and positive scalars ε_1, ε_2, $\cdots$, ε_7 such that the following LMIs are satisfied

$$\left[\begin{array}{cccccccccccc}
M_1 & 0 & M_2 & e^{\alpha h}(A+\alpha I)^T C & he^{\alpha h}\bar{A}^T PE & P & 0 & 0 & 0 & (A+\alpha I)^T & 0 & 0 \\
0 & -Q_1+Q_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
M_2^T & 0 & M_3 & e^{2\alpha h}(B+C)^T C & he^{\alpha h}\bar{B}^T PE & 0 & 0 & e^{\alpha h}C^T P & e^{\alpha h}C^T P & 0 & e^{\alpha h}(B+C)^T & 0 \\
e^{\alpha h}C^T(A+\alpha I) & 0 & e^{2\alpha h}C^T(B+C) & M_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{\alpha h}C^T \\
he^{\alpha h}E^T P\bar{A} & 0 & he^{\alpha h}E^T PB & 0 & -hW & 0 & he^{\alpha h}E^T P & 0 & 0 & 0 & 0 & 0 \\
P & 0 & 0 & 0 & 0 & -\varepsilon_1 I & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & he^{\alpha h}PE & 0 & -h\varepsilon_2 I & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & e^{\alpha h}PC & 0 & 0 & 0 & 0 & -\varepsilon_3 I & 0 & 0 & 0 & 0 \\
0 & 0 & e^{\alpha h}PC & 0 & 0 & 0 & 0 & 0 & -\varepsilon_4 I & 0 & 0 & 0 \\
A+\alpha I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_5 I & 0 & 0 \\
0 & 0 & e^{\alpha h}(B+C) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_6 I & 0 \\
0 & 0 & 0 & e^{\alpha h}C & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_7 I
\end{array}\right] < 0 \qquad (6)$$

$$\begin{bmatrix} -\rho X + e^{2\alpha h} C^T X C & he^{\alpha h} E^T X \\ he^{\alpha h} XE & -X \end{bmatrix} < 0 \qquad (7)$$

where

$$M_1 = P\bar{A} + \bar{A}^T P + 3\beta_0^2 U + (A+\alpha I)^T (A+\alpha I) + Q + hW \quad (8a)$$

$$M_2 = P\bar{B} - e^{\alpha h}\bar{A}^T PC + e^{\alpha h}(A+\alpha I)^T (B+C) \quad (8b)$$

$$M_3 = -Q + \varepsilon_3 \bar{B}^T \bar{B} + 3e^{2\alpha h}(\beta_1+\beta_2)^2 U + e^{2\alpha h}(B+C)^T (B+C) \quad (8c)$$

$$M_4 = -I + e^{2\alpha h}(C^T C + 3\beta_2^2 U) \quad (8d)$$

$$U = (1 + \varepsilon_1 + h\varepsilon_2 + \varepsilon_4 + \varepsilon_5 + \varepsilon_6 + \varepsilon_7) I \quad (8e)$$

$$\bar{A} = A + \alpha I + e^{\alpha h}E,\ \bar{B} = e^{\alpha h}(B + C - E) \quad (8f)$$

Proof: Let $x(t) = e^{-\alpha t} z(t)$, where $\alpha > 0$. Thus, from (1), we have

$$\dot{z}(t) = (A + \alpha I)z(t) + e^{\alpha h}(B + C)z(t-h) + e^{\alpha h}C\dot{z}(t-h) + \bar{f}(z(t), z(t-h), \dot{z}(t-h)) \quad (9)$$

where

$$\left\|\bar{f}(z(t),\ z(t-h),\ \dot{z}(t-h))\right\| = \left\| e^{\alpha t} f(x(t),\ x(t-h),\ \dot{x}(t-h)) \right\| \le \beta_0\|z(t)\| + e^{\alpha h}(\beta_1 + \beta_2)\|z(t-h)\| + e^{\alpha h}\beta_2\|\dot{z}(t-h)\| \quad (10)$$

Furthermore, the system (9) can be written as

$$\frac{d}{dt}[z(t)+e^{\alpha h}E\int_{t-h}^{t}z(s)ds-e^{\alpha h}Cz(t-h)])$$

$$=\overline{A}z(t)+\overline{B}z(t-h)+\overline{f}(z(t),\ z(t-h),\dot{z}(t-h) \qquad (11)$$

Choose the Lyapunov-Krasovskii functional for system (11) as

$$V(z(t))=D^T(z_t)PD(z_t)+\int_{t-\delta h}^{t}z^T(s)Q_1z(s)ds\int_{t-h}^{t-\delta h}z^T(s)Q_2z(s)ds$$

$$+\int_{t-h}^{t}\dot{z}^T(s)\dot{z}(s)ds+\int_{t-h}^{t}\int_{s}^{t}z^T(\theta)Wz(\theta)d\theta ds \quad (12)$$

The time derivative of $V(z(t))$ along the trajectory of system (11) is given by

$$\begin{aligned}\dot{V}(z(t))=&2D^T(z_t)P[\overline{A}z(t)+\overline{B}z(t-h)+\overline{f}(z(t),z(t-h),\dot{z}(t-h))]\\&+z^T(t)Q_1z(t)-z^T(t-\delta h)Q_1z(t-\delta h)\\&+z^T(t-\delta h)Q_2z(t-\delta h)-z^T(t-h)Q_2z(t-h))\\&+\dot{z}^T(t)\dot{z}(t)-\dot{z}^T(t-h)\dot{z}(t-h)\\&+hz^T(t)Wz(t)-\int_{t-h}^{t}z^T(s)Wz(s)ds \qquad (13)\end{aligned}$$

Applying the following well-known inequality to (13)

$$\pm 2a^Tb\le\varepsilon^{-1}a^Ta+\varepsilon b^Tb \qquad (14)$$

for any vectors $a,\ b\in R^n$ and scalar $\varepsilon>0$, we obtain

$$\begin{aligned}\dot{V}(z(t))\le\int_{t-h}^{t}\frac{1}{h}\{&z^T(t)[P\overline{A}+\overline{A}^TP+\varepsilon_1^{-1}PP+3\beta_0^2U\\&+(1+\varepsilon_5^{-1})(A+\alpha I)^T(A+\alpha I)+Q_1+hW]z(t)\\&+2z^T(t)[P\overline{B}+e^{\alpha h}(A+\alpha I)^T(B+C)\\&-e^{\alpha h}\overline{A}^TPC]z(t-h)\\&+2e^{\alpha h}z^T(t)(A+\alpha I)^TC\dot{z}(t-h)\\&+2e^{2\alpha h}z^T(t-h)(B+C)^TC\dot{z}(t-h)\\&+2he^{\alpha h}z^T(s)[E^TP\overline{A}z(t)+E^TP\overline{B}z(t-h)]\\&-z^T(t-\delta h)(Q_1-Q_2)z(t-\delta h)\\&-z^T(t-h)\{Q_2-\varepsilon_3\overline{B}^T\overline{B}\\&-e^{2\alpha h}[3(\beta_1+\beta_2)^2U+(\varepsilon_3^{-1}+\varepsilon_4^{-1})C^TPPC]\\&-e^{2\alpha h}(1+\varepsilon_6^{-1})(B+C)^T(B+C)\}z(t-h)\\&-\dot{z}^T(t-h)[I-3e^{2\alpha h}\beta_2^2U\\&-e^{2\alpha h}(1+\varepsilon_7^{-1})C^TC]\dot{z}(t-h)\\&-z^T(s)(hW-h\varepsilon_2^{-1}e^{2\alpha h}E^TPPE)z(s)\}ds\end{aligned}$$

$$=\int_{t-h}^{t}\frac{1}{h}[z^T(t)\ z^T(t-\delta h)\ z^T(t-h)\ \dot{z}^T(t-h)\ z^T(s)]H$$

$$\times\left[z^T(t)\ z^T(t-\delta h)\ z^T(t-h)\ \dot{z}^T(t-h)\ z^T(s)\right]^Tds(15)$$

where

$$H=\begin{bmatrix}M_1+\varepsilon_1^{-1}PP+\varepsilon_5^{-1}(A+\alpha I)^T(A+\alpha I) & 0 & M_2 & e^{\alpha h}(A+\alpha I)^TC & he^{\alpha h}\overline{A}^TPE\\ 0 & -Q_1+Q_2 & 0 & 0 & 0\\ M_2^T & 0 & M_3+e^{2\alpha h}(\varepsilon_3^{-1}+\varepsilon_4^{-1})C^TPPC+e^{2\alpha h}\varepsilon_6^{-1}(B+C)^T(B+C) & e^{2\alpha h}(B+C)^TC & he^{\alpha h}\overline{B}^TPE\\ e^{\alpha h}C^T(A+\alpha I) & 0 & e^{2\alpha h}C^T(B+C) & M_4+e^{2\alpha h}\varepsilon_7^{-1}C^TC & 0\\ he^{\alpha h}E^TP\overline{A} & 0 & he^{\alpha h}E^TP\overline{B} & 0 & -hW+h\varepsilon_2^{-1}e^{2\alpha h}E^TPPE\end{bmatrix}$$

and M_1,M_2,M_3,M_4 are defined in (8a), (8b), (8c), (8d), respectively. It is easy to see that $\dot{V}(z(t))<0$ if $H<0$. Obviously, $H<0$ if and only if (6) holds. From (7) and lemma 1, the operator $D(z_t)$ is stable. Therefore, we conclude that the system (9) and the system (11) are both asymptotically stable, i.e., the system (1) is exponentially stable with the convergence rate α. Thus, the proof is completed.

3. Numerical Example

Example 1: Consider the following neutral time-delay system with nonlinear disturbances

$$\dot{x}(t)=\begin{bmatrix}-3&0\\0&-5\end{bmatrix}x(t)+\begin{bmatrix}0&0.4\\0.4&0\end{bmatrix}x(t-h)+\begin{bmatrix}0.1&0\\0&0.1\end{bmatrix}\dot{x}(t-h)$$

$$+f(x(t),x(t-h),\dot{x}(t-h)) \qquad (16)$$

where

$$\|f(x(t),x(t-h),\dot{x}(t-h))\|\le 0.1\|x(t)\|+0.2\|x(t-h)\|+0.5\|\dot{x}(t-h)\|.$$

First, we shall compare our delay-dependent stability criterion with those in [7,8,9,10] for checking the asymptotic stability of the system (16). Applying Theorem 1 with $\alpha=0$, by setting $\rho=0.5$ and choosing $E=C$, using the Matlab LMI Toolbox, it is found that the system (16) is asymptotically stable for any constant time delay $h\le 2.5238$. If we choose $E=0.1C$, the upper bound of time delay h is 3.9971. On the other hand, the maximum values of time delay h obtained in [7,8,9,10] for guaranteeing the asymptotic stability of the system (16) are 0.3871, 0.5135, 0.7168, and 1.0732, respectively. Thus, for this example, the delay-dependent stability criterion of this paper gives a less conservative result than those obtained by the methods in [7,8,9,10]. In addition, this example also shows that choosing an appropriate parameter E can maximize the allowable delay bound for guaranteeing the asymptotic stability of the above system.

Next, we consider the effect of the time delay h on the convergence rate α. Again applying Theorem 1, we can find the fact that the convergence rate α decreases when the time delay h increases (i.e. $h=0.2$, $\alpha=0.5021$; $h=0.8$, $\alpha=0.1328$; $h=1.5$, $\alpha=0.0617$; $h=1.8$, $\alpha=0.0214$; $h=2.5$, $\alpha=0.0093$). Furthermore, the result of [11] guarantees the exponential stability of system (16) with time delay $h=0.2$ when convergence rate $\alpha=0.0138$. Hence, the Theorem 1 in this paper significantly improves the result of [11].

4. Conclusion

This paper considers the stability problem for neutral time-delay systems with nonlinear disturbances. Based on the new Lyapunov-Krasovskii functional approach and a parameterized model transformation technique, a less conservative delay-dependent robust exponential stability condition is established. By comparing the proposed result with the recent published papers through a numerical example, it is shown that the derived criterion is less conservative than several recent results.

5. References

[1] S. I. Niculescu, "On delay-dependent stability under model transformations of some neutral linear systems," *Int. J. Control*, vol. 74, pp. 609-617, 2001.

[2] J. K. Hale, and S. M. Verduyn Lunel, *Introduction to functional differential equations*, Springer-Verlag, New York, 1993.

[3] J. D. Chen, C. H. Lien, K. K. FAN, and J. S, Cheng, "Delay-dependent stability criterion for neutral time-delay systems," *Electronics Letters*, vol. 36, pp. 1897-1898, 2000.

[4] G. D. Hui, and G. D. Hu, "Simple criteria for stability of neutral systems with multiple delays," *Int. J. Syst. Sci.*, vol. 28, pp. 1325-1328, 1997.

[5] C. H. Lien, "Asymptotic criterion for neutral systems with multiple time delays," *Electronics Letters*, vol. 35, pp. 850-852, 1999.

[6] S. I. Niculescu, C, E. DE Souza, L. Dugard, and J, M. DION, "Robust exponential stability of uncertain systems with time-varying delays," *IEEE Trans. Automat. Control*, vol. 43, pp. 743-748, 1998.

[7] Q. L. Han, and L. Yu, "Robust stability of linear neutral systems with nonlinear parameter perturbations," *IEE Proc.-Control Theory Appl.*, vol. 151, pp. 539-546, 2004.

[8] W. A. Zhang, and L. Yu, "Delay-dependent robust stability of neutral systems with mixed delays and nonlinear perturbations," *Acta Automatica Sinica*, vol. 33, pp.863-866, 2007.

[9] F. Qiu, B. Cui, and Y. Ji, "A delay-dividing approach to stability of neutral system with mixed delays and nonlinear perturbations," *Applied Mathematical Modelling*, vol. 34, pp.3701-3707, 2010.

[10] R. Rakkiyappan, P. Balasubramaniam, and R. Krishnasamy, "Delay dependent stability analysis of neutral systems with mixed time-varying delays and nonlinear perturbations," *Journal of Computational and Applied Mathematics*, vol. 235, pp. 2147-2156, 2011.

[11] M. Syed Ali, "On exponential stability of neutral delay differential system with nonlinear uncertainties," *Communications in Nonlinear Science and Numerical Simulation*, vol. 17, pp. 2595-2601, 2012.

常數延遲非線性擾動中立系統之指數收斂分析

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摘要

本文旨在研究常數延遲非線性擾動中立系統之延遲相關指數收斂問題。藉由李亞普諾-克羅斯威斯基泛函數方法與參數模型轉換技巧，針對上述系統，提出線性矩陣不等式指數穩定測試準則。舉例證實本研究方法明顯改善文獻結果。

關鍵字： 中立系統，指數收斂，非線性擾動，參數模型轉換。