離散式大型延遲系統之線性矩陣不等式強健穩定準則

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摘要

本文旨在探討離散式大型延遲系統之漸近式與強健性穩定度測試問題。藉由離散式奇異模型轉換策略、擴展型李亞普諾-克羅斯威斯基泛函數及有限總和不等式方法，針對上述系統，提出新時延相關之強健穩定準則。文中所提準則也可用於具有常數時間延遲之單一系統的漸近與強健穩定度測試。本文之主要特點是所提出之準則表示為具有較少變數之線性矩陣不等式形式，可便於 MATLAB 軟體之 LMI 工具箱求解，並獲得最佳結果及減少求解之計算時間。最後，舉例證實本研究方法明顯改善現有文獻結果。

關鍵字：離散式大型延遲系統，強健性穩定度，有限總和不等式方法，線性矩陣不等式。

1. 前言

一般而言，大型系統可視為許多子系統之組合，譬如傳輸系統、電力系統、通訊系統等。多年來，針對大型連續系統之穩定度分析與控制器設計問題，受到廣泛的討論。然而，其研究結果是採用不同方法(包括矩陣量度之性質、比較定理與傳統李亞普諾函數)，得到較為保守的穩定測試條件與控制準則[1-12]。另外，離散式大型系統之穩定度與控制研究結果則相當少，且這些定理是狀態延遲無關[13,14,15]。事實上，當時間延遲很小時，延遲無關之穩定測試條件是較延遲相關之穩定測試條件保守。因此，針對具有不定參數擾動與常數時間延遲之大型互連離散系統，本文將應用線性矩陣不等式策略，以推導新時延相關之強健穩定測試條件，來降低文獻結果的保守性。

2. 離散式大型延遲系統之漸近穩定準則

考慮由 N 個互相連接之離散延遲子系統 S_i ($i=1,2,\cdots,N$)所組成之大型互連離散延遲系統，其第 i 個離散延遲子系統描述如下

$$x_i(k+1)=A_{ii}x_i(k)+B_{ii}x_i(k-h)+\sum_{\substack{j=1\\ j\neq i}}^{N}[A_{ij}x_j(k)+B_{ij}x_j(k-h)] \quad (1a)$$

$$x_i(k)=\phi_i(k),\quad k=-h,-h+1,\cdots,0 \quad (1b)$$

其中 $x_i(k)\in R^{n_i}$ 是狀態向量；A_{ii}，B_{ii}，A_{ij}，B_{ij} $(i,j=1,2,\cdots,N)$ 是具有適當維度之常數矩陣。h 表示正整數時間延遲項。$\phi_i(k)$ 為在時間 k 之初始函數。

組合 N 個子系統，可得全系統如下

$$x_f(k+1)=A_f x_f(k)+B_f x_f(k-h) \quad (2a)$$

$$x_f(k)=\phi_f(k),\quad k=-h,-h+1,\cdots,0 \quad (2b)$$

$$x_f(k)=[x_1^T(k)\ x_2^T(k),\cdots,x_N^T(k)]^T \quad (2c)$$

$$x_f(k+1)=[x_1^T(k+1)\ x_2^T(k+1),\cdots,x_N^T(k+1)]^T \quad (2d)$$

其中

$$A_f=\begin{bmatrix}A_{11}&A_{12}&\cdots&A_{1N}\\A_{21}&A_{22}&\cdots&A_{2N}\\\vdots&\vdots&\ddots&\vdots\\A_{N1}&A_{N2}&\cdots&A_{NN}\end{bmatrix} \quad (3a)$$

$$B_f=\begin{bmatrix}B_{11}&B_{12}&\cdots&B_{1N}\\B_{21}&B_{22}&\cdots&B_{2N}\\\vdots&\vdots&\ddots&\vdots\\B_{N1}&B_{N2}&\cdots&B_{NN}\end{bmatrix} \quad (3b)$$

補助定理一[16]

對任何向量 $v_1,v_2\in R^n$ 及任意對稱正定矩陣 M，則下列不等式成立

$$2v_1^T v_2\le v_1^T M v_1+v_2^T M^{-1}v_2 \quad (4)$$

補助定理二[17]

令 $y(k)=x(k+1)-x(k)(k=1,2,\cdots)$，其中 $x(k)\in R^n$。若

存在任意正整數 d 及對稱正定矩陣 Φ，則下列不等式成立

$$-d\sum_{\theta=k-d}^{k-1} y^{\mathrm{T}}(k)\Phi y(k) \le \begin{bmatrix} x(k) \\ x(k-d)\end{bmatrix}^{\mathrm{T}} \begin{bmatrix} -\Phi & \Phi \\ \Phi & -\Phi \end{bmatrix} \begin{bmatrix} x(k) \\ x(k-d)\end{bmatrix} \quad (5)$$

定理一

若存在對稱正定矩陣 P_1，P_2，P_3，Q，M_1，M_2，M_3 及實數矩陣 W，使得下列條件成立

$$\begin{bmatrix} P_2(A_f+B_f-I)+(A_f+B_f-I)^{\mathrm{T}}P_2+Q-M_2+2WM_2-W & -M_2 & P_1+(A_f+B_f-I)^{\mathrm{T}}P_3-P_2 & W & hP_2B_f \\ M_2^{\mathrm{T}}-W^{\mathrm{T}} & & 0 & 0 & 0 \\ P_1+P_3(A_f+B_f-I)-P_2 & 0 & -2P_3+P_1+hM_1+h^2(M_2+M_3) & 0 & hP_3B_f \\ W^{\mathrm{T}} & 0 & 0 & -M_3 & 0 \\ hB_f^{\mathrm{T}}P_2 & 0 & hP_f^{\mathrm{T}}B_f & 0 & -hM_1 \end{bmatrix} < 0 \quad (6)$$

則系統(1)式為漸近穩定，其中 A_f 與 B_f 分別定義於(3a)式與(3b)式。

證明

藉由系統參數分解方法，則系統(2)式變成如下

$$x_f(k+1) = x_f(k) + y_f(k) \quad (7a)$$

$$x_f(k+1) = (A_f + B_f)x_f(k) - B_f\sum_{s=k-h}^{k-1}[x_f(s+1)-x_f(s)] = (A_f+B_f)x_f(k) - B_f\sum_{s=k-h}^{k-1} y_f(s) \quad (7b)$$

再者，根據離散式奇異模型轉換，則(7)式改寫如下

$$x_f(k+1) = x_f(k) + y_f(k) \quad (8a)$$

$$0 = (A_f + B_f - I)x_f(k) - y_f(k) - B_f\sum_{s=k-h}^{k-1} y_f(s) \quad (8b)$$

選取 Lyapunov-Krasovskii 泛函數如下

$$V(k) = V_1(k)+V_2(k)+V_3(k) \quad (9)$$

其中

$$V_1(k) = [x_f^{\mathrm{T}}(t) \quad y_f^{\mathrm{T}}(t)]\begin{bmatrix} I & 0 \\ 0 & 0\end{bmatrix}\begin{bmatrix} P_1 & 0 \\ P_2 & P_3\end{bmatrix}\begin{bmatrix} x_f(t) \\ y_f(t)\end{bmatrix} \quad (10a)$$

$$V_2(k) = \sum_{\theta=-h+1}^{0}\sum_{s=k-1+\theta}^{k-1} y_f^{\mathrm{T}}(s)[M_1+h(M_2+M_3)]y_f(s) \quad (10b)$$

$$V_3(k) = \sum_{s=k-h}^{k-1} x_f^{\mathrm{T}}(s)Qx_f(s) \quad (10c)$$

將(9)式沿系統(8)式的軌跡，並取順向差分(forward difference)及引用文獻[17]的方法，可得

$$\begin{aligned} \Delta V(k) &= 2x_f^{\mathrm{T}}(k)P_1y_f(k) + x_f^{\mathrm{T}}(k)Qx_f(k) \\ &\quad + y_f^{\mathrm{T}}(k)[P_1+hM_1+h^2(M_2+M_3)]y_f(k) \\ &\quad - x_f^{\mathrm{T}}(k-h)Qx_f(k-h) \\ &\quad - \sum_{s=k-h}^{k-1} y_f^{\mathrm{T}}(s)[M_1+h(M_2+M_3)]\,y_f(s) \\ &\quad + 2x_f^{\mathrm{T}}(k)[x_f(k)-x_f(k-h)\sum_{s=k-h}^{k-1} y_f(s)] \\ &= 2[x_f^{\mathrm{T}}(k) \quad y_f^{\mathrm{T}}(k)]\begin{bmatrix} P_1 & P_2 \\ 0 & P_3\end{bmatrix} \\ &\quad \times \begin{bmatrix} y_f(k) \\ [A_f+B_f-I]x_f(k)-y_f(k)-\sum_{s=k-h}^{k-1} B_fy_f(s)\end{bmatrix} \\ &\quad + x_f^{\mathrm{T}}(k)Qx_f(k)+y_f^{\mathrm{T}}(k)[P_1+hM_1+h^2(M_2+M_3)]y_f(k) \\ &\quad - x_f^{\mathrm{T}}(k-h)Qx_f(k-h) \\ &\quad - \sum_{s=k-h}^{k-1} y_f^{\mathrm{T}}(s)[M_1+h(M_2+M_3)]\,y_f(s) \\ &\quad + 2x_f^{\mathrm{T}}(k)[x_f(k)-x_f(k-h)\sum_{s=k-h}^{k-1} y_f(s)] \end{aligned} \quad (11)$$

引用補助定理一，可得

$$-\sum_{s=k-h}^{k-1} 2[x_f^{\mathrm{T}}(k) \quad y_f^{\mathrm{T}}(k)]\begin{bmatrix} P_1 & P_2 \\ 0 & P_3\end{bmatrix}\begin{bmatrix} 0 \\ B\end{bmatrix}y_f(s) \le h[x_f^{\mathrm{T}}(k)\, y_f^{\mathrm{T}}(k)]\begin{bmatrix} P_2B \\ P_3B\end{bmatrix}M_1^{-1}\begin{bmatrix} B^{\mathrm{T}}P_2 & B^{\mathrm{T}}P_3\end{bmatrix}\begin{bmatrix} x_f(k) \\ y_f(k)\end{bmatrix} + \sum_{s=k-h}^{k-1} y_f^{\mathrm{T}}(s)M_1y_f(s) \quad (12)$$

應用補助定理二(有限總和不等式方法)，可得

$$-\sum_{s=k-h}^{k-1} h\,y_f^{\mathrm{T}}(s)M_2y_f(s) \le -[x_f(k)-x_f(k-h)]^{\mathrm{T}}M_2[x_f(k)-x_f(k-h)] \quad (13a)$$

$$-\sum_{s=k-h}^{k-1} h\,y_f^{\mathrm{T}}(s)M_3y_f(s) \le -\left(\sum_{s=k-h}^{k-1} y_f(s)\right)^{\mathrm{T}} M_3\left(\sum_{s=k-h}^{k-1} y_f(s)\right) \quad (13b)$$

將(12)式及(13)式代入(11)式，可得

$$\Delta V(k)\le[x_f^{\mathrm{T}}(k)x_f^{\mathrm{T}}(k-h)y_f^{\mathrm{T}}(k)\sum_{s=k-h}^{k-1}y_f(s)]\Omega\begin{bmatrix}x_f(k)\\x_f(k-h)\\y_f(k)\\\sum_{s=k-h}^{k-1}y_f(s)\end{bmatrix}\quad(14)$$

其中

$$\Omega=\begin{bmatrix}\Omega_1 & M_2-W & \Omega_2 & W\\ M_2^{\mathrm{T}}-W^{\mathrm{T}} & -M_2 & 0 & 0\\ \Omega_2^{\mathrm{T}} & 0 & \Omega_3 & 0\\ W^{\mathrm{T}} & 0 & 0 & -M_3\end{bmatrix}\quad(15a)$$

$$\Omega_1=P_2(A_f+B_f-I)+(A_f+B_f-I)^{\mathrm{T}}P_2+Q$$

$$+P_2B_fM_1^{-1}B_f^{\mathrm{T}}P_2-M_2+2W\quad(15b)$$

$$\Omega_2=P_1+(A_f+B_f-I)^{\mathrm{T}}P_3-P_2+P_2B_fM_1^{-1}B_f^{\mathrm{T}}P_3\quad(15c)$$

$$\Omega_3=-2P_3+P_1+hM_1+h^2(M_2+M_3)+P_3B_fM_1^{-1}B_f^{\mathrm{T}}P_3\quad(15d)$$

由上述分析得知，若 $\Omega<0$ 成立，則 $\Delta V(k)<0$。再者，根據 Schur complement 觀念[18]，若(6)式成立，則 $\Omega<0$。因此，可確定系統(8)式為漸近穩定，亦即保證兩系統(1)式及(2)式均為漸近穩定。故得證。

3. 不確定性大型離散系統之強健穩定準則

考慮由 N 個互相連接之不確定性離散延遲子系統 S_i $(i=1,2,\cdots,N)$ 所組成之不確定性大型互連離散延遲系統，其第 i 個不確定性離散延遲子系統描述如下

$$x_i(k+1)=[A_{ii}+\Delta A_{ii}(k)]x_i(k)+[B_{ii}+\Delta B_{ii}(k)]x_i(k-h)$$

$$+\sum_{\substack{j=1\\j\neq i}}^{N}\{[A_{ij}+\Delta A_{ij}(k)]x_j(k)+[B_{ij}+\Delta B_{ij}(k)]x_j(k-h)\}\quad(16a)$$

$$x_i(k)=\phi_i(k),\quad k=-h,-h+1,\cdots,0\quad(16b)$$

其中 $x_i(k)\in R^{n_i}$ 為狀態向量；A_{ii}，B_{ii}，A_{ij}，B_{ij} $(i,j=1,2,\cdots,N)$ 是具有適當維度之常數矩陣。h 表示正整數時間延遲項。$\phi_i(k)$ 為在時間 k 之初始函數。$\Delta A_{ii}(k)$，$\Delta B_{ii}(k)$，$\Delta A_{ij}(k)$，$\Delta B_{ij}(k)$ 為參數擾動且其範數是有界的。

組合 N 個子系統，可得全系統如下

$$x_f(k+1)=[A_f+\Delta A_f(k)]x_f(k)+[B_f+\Delta B_f(k)]x_f(k-h)\quad(17a)$$

$$x_f(k)=\phi_f(k),\quad k=-h,-h+1,\cdots,0\quad(17b)$$

$$x_f(k)=[x_1^{\mathrm{T}}(k)\;x_2^{\mathrm{T}}(k),\cdots,x_N^{\mathrm{T}}(k)]^{\mathrm{T}}\quad(17c)$$

$$x_f(k+1)=[x_1^{\mathrm{T}}(k+1)\;x_2^{\mathrm{T}}(k+1),\cdots,x_N^{\mathrm{T}}(k+1)]^{\mathrm{T}}\quad(17d)$$

其中 A_f 與 B_f 分別定義於(3a)式與(3b)式，且

$$\Delta A_f(k)=\begin{bmatrix}\Delta A_{11}(k) & \Delta A_{12}(k) & \cdots & \Delta A_{1N}(k)\\ \Delta A_{21}(k) & \Delta A_{22}(k) & \cdots & \Delta A_{2N}(k)\\ \vdots & \vdots & \ddots & \vdots\\ \Delta A_{N1}(k) & \Delta A_{N2}(k) & \cdots & \Delta A_{NN}(k)\end{bmatrix}\quad(18a)$$

$$\Delta B_f(k)=\begin{bmatrix}\Delta B_{11}(k) & \Delta B_{12}(k) & \cdots & \Delta B_{1N}(k)\\ \Delta B_{21}(k) & \Delta B_{22}(k) & \cdots & \Delta B_{2N}(k)\\ \vdots & \vdots & \ddots & \vdots\\ \Delta B_{N1}(k) & \Delta B_{N2}(k) & \cdots & \Delta B_{NN}(k)\end{bmatrix}\quad(18b)$$

再者，$\Delta A_f(k)$ 與 $\Delta B_f(k)$ 分別滿足下列條件

$$\Delta A_f(k)=E_1F_1(k)H_1,\quad\Delta B_f(k)=E_2F_2(k)H_2\quad(19)$$

其中 E_1，E_2，H_1，H_2 為具有適當維度的已知常數矩陣；$F_i(k)$ 為具有適當維度的未知矩陣且滿足下列條件

$$F_i^{\mathrm{T}}(k)F_i(k)\le I,\quad i=1,2\quad(20)$$

補助定理三[16]

令 E，F 及 H 為具有適當維度之實數向陣且 $\|F\|\le 1$，則下列不等式成立

$$EFH+H^{\mathrm{T}}F^{\mathrm{T}}E^{\mathrm{T}}\le\gamma^{-1}EE^{\mathrm{T}}+\gamma H^{\mathrm{T}}H\quad(21)$$

定理二

若存在對稱正定矩陣 P_1，P_2，P_3，Q，M_1，M_2 及正純量 α_1，α_2，β_1，β_2，μ_1，μ_2，使得下列條件成立

$$\left[\begin{array}{ccccc}\Psi & M_2 & P_1+(A_f+B_f-I)^{\mathrm{T}}P_3-P_2 & hP_2B_f & P_2E_1\\ M_2 & -M_2 & 0 & 0 & 0\\ P_1+P_3(A_f+B_f-I)-P_2 & 0 & -Y & hP_3B_f & 0\\ hB_f^{\mathrm{T}}P_2 & 0 & hB_f^{\mathrm{T}}P_3 & -hM & 0\\ E_1^{\mathrm{T}}P_2 & 0 & 0 & 0 & -\alpha_1 I\\ E_2^{\mathrm{T}}P_2 & 0 & 0 & 0 & 0\\ hE_2^{\mathrm{T}}P_2 & 0 & 0 & 0 & 0\\ 0 & 0 & E_1^{\mathrm{T}}P_3 & 0 & 0\\ 0 & 0 & E_2^{\mathrm{T}}P_3 & 0 & 0\\ 0 & 0 & E_2^{\mathrm{T}}P_3 & 0 & 0\end{array}\right.$$

$$\left.\begin{array}{ccccc} P_2E_2 & hP_2E_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & P_3E_1 & P_3E_2 & P_3E_1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -\beta_1 I & 0 & 0 & 0 & 0 \\ 0 & -h\mu_1 I & 0 & 0 & 0 \\ 0 & 0 & -\alpha_2 I & 0 & 0 \\ 0 & 0 & 0 & -\beta_2 I & 0 \\ 0 & 0 & 0 & 0 & -\mu_2 I \end{array}\right] < 0 \quad (22)$$

則系統(14)式為漸近穩定，其中

$$\Psi = P_2(A_f + B_f - I) + (A_f + B_f - I)^{\mathrm{T}} P_2 + Q - M_2$$

$$+ (\alpha_1 + \alpha_2) H_1^{\mathrm{T}} H_1 + (\beta_1 + \beta_2) H_2^{\mathrm{T}} H_2 \quad (23a)$$

$$Y = 2P_3 - P_1 - h[M_1 + hM_2 + (\mu_1 + \mu_2) H_2^{\mathrm{T}} H_2] \quad (23b)$$

證明

如同定理一之證明步驟且引用補助定理三，故省略證明過程。因此，可確定兩系統(16)式與(17)式均為漸近穩定。

4. 例題說明

例題一

考慮大型互連離散延遲系統如下

$$x_i(k+1) = A_{ii} x_i(k) + B_{ii} x_i(k-h)$$

$$+ \sum_{\substack{j=1 \\ j \neq i}}^{3} [A_{ij} x_j(k) + B_{ij} x_j(k-h)], \; i = 1,2,3 \quad (24)$$

其中

$$A_{11} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.6 \end{bmatrix}, A_{12} = \begin{bmatrix} -0.1 & 0.02 \\ 0.02 & 0.1 \end{bmatrix}, A_{13} = \begin{bmatrix} 0.2 & 0.03 \\ 0 & -0.2 \end{bmatrix}$$

$$A_{22} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.8 \end{bmatrix}, A_{21} = \begin{bmatrix} 0.2 & 0 \\ 0.04 & 0.1 \end{bmatrix}, A_{23} = \begin{bmatrix} 0.1 & 0.01 \\ 0.01 & 0.1 \end{bmatrix}$$

$$A_{33} = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.9 \end{bmatrix}, A_{31} = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix}, A_{32} = \begin{bmatrix} 0.1 & 0.5 \\ 0.02 & -0.1 \end{bmatrix}$$

$$B_{11} = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix}, B_{12} = \begin{bmatrix} 0.02 & 0.01 \\ 0.01 & 0.01 \end{bmatrix}, B_{13} = \begin{bmatrix} 0.01 & 0.01 \\ 0 & 0 \end{bmatrix}$$

$$B_{22} = \begin{bmatrix} 0 & 0 \\ 0.01 & 0.02 \end{bmatrix}, B_{21} = \begin{bmatrix} 0.02 & 0.02 \\ 0 & 0 \end{bmatrix}, B_{23} = \begin{bmatrix} 0 & 0 \\ 0.02 & 0.02 \end{bmatrix}$$

$$B_{33} = \begin{bmatrix} 0 & 0.02 \\ 0.02 & 0 \end{bmatrix}, B_{31} = \begin{bmatrix} 0 & 0 \\ 0.02 & 0.02 \end{bmatrix}, B_{32} = \begin{bmatrix} 0.02 & 0 \\ 0.02 & 0 \end{bmatrix}$$

本問題是系統(24)式仍保持漸近穩定情況下，能容許多大之時間延遲量 h。

解

引用本文之定理一及 MATLAB LMI Toolbox 軟體，求得當 $0 < h \leq 18$，則保證系統(24)式為漸近穩定。

然而，文獻[13,15]之穩定測試準則無法應用於系統(24)式。因此，本文之定理一的穩定條件較優於文獻[13,15]的穩定條件。

5. 結論

本文乃提出離散式奇異模型轉換策略及有限總和不等式方法，應用於大型互連離散延遲系統之漸近式與強健性穩定度研究，而本方法的特點可歸納如下：

(一)藉由大型互連離散延遲系統之離散式奇異模型轉換技巧，促使等效大型互連系統之穩定度易於分析。

(二) 選取擴展型李亞普諾-克羅斯威斯基泛函數，並引用有限總和不等式方法，可獲得大型互連離散延遲系統之強健穩定度之最大時間延遲估測值。

(三)根據 Schur complement 觀念與線性矩陣不等式技巧，推導出非保守之時延相關強健穩定準則，且便於 MATLAB 軟體之 LMI 工具箱模擬。

(四)改善目前文獻之時延無關穩定測試條件，未來將進一步應用於具有區間時變時延之大型離散隨機系統的漸近與指數穩定度測試問題。

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An LMI-Based Robust Stability Criterion for Discrete-Time Large-Scale Delay Systems

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Abstract

This paper investigates the problems of asymptotic and robust stability for discrete-time large-scale delay systems. By means of discrete-time singular model transformation strategy, an augmented Lyapunov-Krasovskii functional and finite sum inequality approach, a new delay-dependent criterion is derived in order to guarantee the robust stability of the above systems. The proposed stability criterion is also used for testing the asymptotic and robust stability of single system, and is expressed in terms of the few-variable based linear matrix inequality (LMI), which is convenient to get the optimal stability result and can reduce the computational time via LMI toolbox in MATLAB software. Finally, an example is used to show the less conservative result of the proposed approach compared with the previous one.

Keywords: Discrete-time large-scale delay systems, robust stability, finite sum inequality approach, linear matrix inequality (LMI).