Analysis of Nonlinear Dynamic Behavior and Pull-in prediction of Micro Circular Plate Actuator

Chin-Chia Liu [1], Hao-Kai Wu [1]

[1] Department of Industrial Education and Technology, National Changhua University of Education, Bao-Shan Campus: Number 2, Shi-Da Road, Changhua, 500, Taiwan, R.O.C.

NSC Project：NSC 101-2221-E-018 -007 -

Abstract

A hybrid differential transformation and finite difference scheme is used to analyze the complex nonlinear behavior of an electrostatically-actuated micro circular plate devices which is not easily analyzed using traditional methods such as perturbation theory or Galerkin approach method due to the complexity of the interactions among the electrostatic coupling effect, the residual stress and the nonlinear electrostatic force. The numerical results for the pull-in voltage are found to deviate by no more than 0.27% from the literature using various computational methods. Thus, the basic validity of the hybrid numerical scheme is confirmed. In the present study, the hybrid numerical scheme is applied to analyze the dynamic behavior of a micro circular plate actuated by pure DC or combined DC / AC loading schemes. The micro circular plate displacement is analyzed as a function of both the magnitude and the frequency of the ac voltage. The effects of the residual stress and initial gap height on the pull-in voltage of the micro circular plate are systematically explored.

Keywords：Micro circular plate, Pull-in voltage, Hybrid method; Differential transformation, Finite difference method.

1. Introduction

Micro-electro-mechanical systems (MEMS) devices have found widespread use throughout industry for such applications as accelerometers and pressure sensors, micro-scale actuators, electrostatic rotary comb actuators, and so on. The electrostatic actuation is the most commonly preferred due to its simplicity and high efficiency [1]. The literature contains many investigations into the pull-in phenomenon. Hung and Senturia [2] examined the feasibility of using the leveraged bending and strain-stiffening methods to extend the travel range of analog-tuned electrostatic actuators prior to pull-in. Gang and Kai-Tak [3] performed an analytical investigation into the dynamic behavior of clamped one-dimensional rectangular and two-dimensional axi-symmetric films. The results showed that the behavior of both films was critically dependent on the ratio between the film-lower electrode gap and the film thickness.

Various researchers have investigated the use of hybrid DC / AC schemes in driving electrostatically-actuated MEMS devices [4-7]. Younis *et al* [5] proposed a novel RF MEMS switch actuated using a combined DC / AC loading scheme. It was shown that an appropriate specification of the magnitude and frequency of the AC voltage enabled the driving voltage required to induce the pull-in event to be significantly reduced. Younis [6] examined the dynamic behavior of micro-beams subject to combined DC / AC loading and derived analytical expressions for the micro-beam motion under primary resonance conditions. In general, MEMS devices with a circular plate configuration have a better structure flexibility than those with a rectangular plate configuration due to a lack of sharp edges and a lower residual stress after multiple depositions [7]. Differential transformation theory was originally proposed by Zhao as a means of solving linear and nonlinear initial value problems in the circuit analysis domain.

However, in more recent years, researchers have extended its use to the analysis of a variety of initial value problems in the mechanical engineering field. Chen *et al.* [8] demonstrated that the hybrid differential transformation and finite difference method provides a precise and computationally-efficient means of analyzing the nonlinear dynamic behavior of fixed-fixed micro-beams.The same group also used the hybrid method to analyze the nonlinear dynamic response of an electrostatically-actuated micro system subject to the effects of residual stress and a uniform hydrostatic pressure acting on the upper surface [9-11]. A numerical investigation was performed into the entropy generated within a mixed convection flow with viscous dissipation effects in a parallel-plate vertical channel using differential transformation method by Chen et al. [12]. In the present study, the hybrid differential transformation and finite difference method is applied to analyze the dynamic behavior of a micro circular plate actuated by pure DC or combined DC / AC loading schemes. The analysis takes account of the axial residual stress within the micro circular plate and explores the dynamic response of the plate as a function of the magnitude of the AC driving voltage.

2. Differential Transformation Theory

This section introduces the basic principles of the differential transformation method. Let $x(t)$ be an analyzable function in the time domain T. The differential transformation of x at time $t=t_0$ in the K domain is defined as

$$X(k;t_0)=M(k)\left(\frac{d^k}{dt^k}(q(t)x(t))\right)_{t=t_0} \quad , \quad k\in K \qquad (1)$$

where k belongs to a set of non-negative integers which collectively form the K domain ; $X(k;t_0)$ is the transformed function in the transformation domain (otherwise referred to as the spectrum of $x(t)$ at $t=t_0$ in the K domain) ; $M(k)$ is a weighting factor, and $q(t)$ is a kernel function corresponding to $x(t)$. Note that $M(k)$ and $q(t)$ are both non-zero and $q(t)$ is analyzable in T.

The inverse differential transformation of $X(k;t_0)$ is given as

$$x(t)=\frac{1}{q(t)}\sum_{k=0}^{\infty}\frac{(t-t_0)^k}{k!}\frac{X(k;t_0)}{M(k)} \quad , \quad t\in T \qquad (2)$$

where $M(k)=H^k/k!$ and $q(t)=1$. Note that H is the time interval.

Let $t_0=0$. Equation (1) then becomes

$$X(k)=\frac{H^k}{k!}\left[\frac{d^k x(t)}{dt^k}\right]_{t=0} \quad , \quad k\in K \qquad (3)$$

The inverse differential transformation of $X(k)$ can then be expressed as

$$x(t)=\sum_{k=0}^{\infty}\left(\frac{t}{H}\right)^k X(k) \quad , \quad t\in T \qquad (4)$$

Substituting Eq. (3) into Eq. (4) gives

$$x(t)=\sum_{k=0}^{\infty}\frac{t^k}{k!}\left[\frac{d^k x(t)}{dt^k}\right]_{t=0} \quad , \quad t\in T \qquad (5)$$

Equation (5) can be expanded as a Taylor series. The basic operational properties of the differential transformation method can be summarized as follows:

(a) Linearity operation

$$T[\alpha x(t)+\beta y(t)]=\alpha X(k)+\beta Y(k) \qquad (6)$$

where T denotes differential transformation and α and β are any real number.

(b) Convolution operation

$$T[x(t)y(t)]=X(k)\otimes Y(k)=\sum_{\ell=0}^{k}X(\ell)Y(k-\ell) \qquad (7)$$

where $\otimes$ denotes convolution.

(c) Differential operation

$$T\left[\frac{d^n x(t)}{dt^n}\right]=\frac{(k+n)!}{k!H^n}X(k+n) \qquad (8)$$

where n is the order of differentiation

(d) Differential transformation of $\sin(t)$ and $\cos(t)$ functions

$$T[\sin(\alpha t+\beta)]=\frac{(\alpha H)^k}{k!}\sin\left(\frac{\pi k}{2}+\beta\right) \qquad (9)$$

where α and β are any real number. [8-12].

3. Governing Equation of Motion for Micro Circular Plate

In deriving the governing equation of motion for the micro circular plate shown in Fig. 1, the present study considers both the residual stress within the plate and the hydrostatic pressure acting on its upper surface. However, for reasons of simplicity, the squeeze-film damping effect between the two plates is ignored. The governing equation is therefore given as [9]

$$\rho h\frac{\partial^2 w}{\partial t^2}+D\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}\right)\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}\right) -T_r\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}\right)=\frac{\varepsilon_0 V(t)^2}{2(G-w)^2}+S_0 \quad , \qquad (10)$$

where ε_0 , h and G are represent the permittivity of free space, the thickness of the micro circular plate, and the initial gap height between the upper and lower plates, respectively. In addition, $V(t)$ is the voltage between the two plates, (i.e., $V(t)=V_{DC}+V_{AC}\sin(\omega t)$) , ρ is the density of the micro circular plate, and w is the transverse deflection of the micro circular plate at a distance r from the center of the plate. Note that w is a function only of the position r and the time t, i.e., $w=w(r,t)$. In other words, the symmetry transverse deflection of micro circular plate is irrelevant to polar coordinate θ . Finally, S_0 is the uniform hydrostatic pressure acting on the upper surface of the plate, T_r is the residual stress within the plate, and D is the flexural rigidity of the plate, i.e.,

$$D=\frac{Eh^3}{12(1-\upsilon^2)} \qquad (11)$$

where υ and E are the Poisson ratio and Young's modulus of the upper circular plate, respectively.

The boundary conditions for the governing equation of motion of the micro circular plate for are defined as follows:

$$w(r,t)=\frac{\partial u(r,t)}{\partial r}=0 \quad \text{at} \quad r=\pm R \qquad (12)$$

where R is the radius of the micro circular plate. (Note that Eq. (12) i assumes that the plate is clamped at its edges.) Finally, the initial condition is defined as:

$$w(r,0)=\frac{\partial w(r,0)}{\partial t}=0 \qquad (13)$$

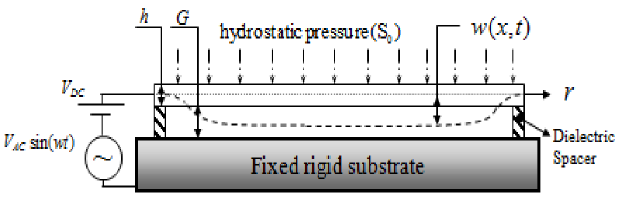

Fig. 1. Schematic illustration showing electrostatic actuation of micro circular plate.

3.1. Application of Hybrid Method to solution of Dimensionless Governing Equation

For analytical convenience, let the transverse displacement of the micro circular plate w be normalized with respect to the initial gap between the electrodes. Furthermore, let the radial distance position r be normalized with respect to the radius of the micro circular plate, and let time t be normalized with respect to a time constant T_n, where T_n is defined as $T_n = \sqrt{\rho h R^4 / D}$. Finally, let the excitation frequency ω be normalized by taking the product of ω and the time constant T_n, i.e.

$$\bar{w} = \frac{w}{G} \ , \ \bar{r} = \frac{r}{R} \ , \ \bar{t} = \frac{t}{T_n} \ , \ \bar{\omega} = \omega T_n \ . \qquad (14)$$

Let the following parameters be defined:

$$\alpha_1 = \frac{\varepsilon_0 R^4}{2DG^3} \ , \ \bar{T_r} = \frac{T_r R^2}{D} \ , \ \bar{S_0} = \frac{S_0 R^4}{DG} \ . \qquad (15)$$

Substituting Eqs. (14) and (15) into Eqs. (10), (12) and (13), the dimensionless governing equation of motion for the micro circular plate is obtained as

$$\frac{\partial^2 \bar{w}}{\partial \bar{t}^2} + \frac{\partial^4 \bar{w}}{\partial \bar{r}^4} + \frac{2}{\bar{r}}\frac{\partial^3 \bar{w}}{\partial \bar{r}^3} - \frac{1}{\bar{r}^2}\frac{\partial^2 \bar{w}}{\partial \bar{r}^2} + \frac{1}{\bar{r}^3}\frac{\partial \bar{w}}{\partial \bar{r}} - \bar{T_r}\frac{\partial^2 \bar{w}}{\partial \bar{r}^2} - \bar{T_r}\frac{1}{\bar{r}}\frac{\partial \bar{w}}{\partial \bar{r}} = \frac{\alpha_1 [V_{DC} + V_{AC}\sin(\bar{\omega}\bar{t})]^2}{(1-\bar{w})^2} + \bar{S_0} \qquad (16)$$

The corresponding dimensionless boundary conditions are given as follows:

$$\bar{w}(\bar{r},\bar{t}) = \frac{\partial \bar{w}(\bar{r},\bar{t})}{\partial \bar{r}} = 0 \quad \text{at} \quad \bar{r} = \pm 1 \ . \qquad (17)$$

Finally, the initial condition is equal to

$$\bar{w}(\bar{r},0) = \frac{\partial \bar{w}(\bar{r},0)}{\partial \bar{t}} = 0 \qquad (18)$$

the nonlinear governing equation of motion for the micro circular plate (Eq. (16)) is solved using the hybrid differential transformation and finite difference method described in [8-10]. The solution procedure commences by discretizing the equation of motion with respect to the time domain t using the differential transformation method, and the transverse displacement of the micro plate is then discretized spatially in the radial direction using the finite difference approximation method based on fourth-order and second-order accurate central difference formulae.

4. Numerical Results and Discussion

In this section, the validity of the proposed hybrid computational scheme in modeling the dynamic response of the micro circular actuator shown in Fig. 1 is demonstrated by comparing the predicted value of the pull-in voltage with that obtained using various schemes in the literature. Note that the hydrostatic pressure acting on the upper surface of the moveable electrode is ignored when applying the hybrid numerical method in order to enable a fair comparison to be made with the results presented in the literature. In performing the comparison, two different models of the micro circular actuator are considered (see Table 1).

As shown in Table 2, the pull-in voltage of the Model 1 micro circular plate is found to be 363 V when using the hybrid differential transformation and finite difference method. The equivalent values of the pull-in voltage computed using Osternberg's model [7] and the reduced-order model [7] are 362 V and 364 V, respectively. In other words, the predicted value of the pull-voltage obtained using the hybrid method deviates by just 0.27% from the results presented in [7]. In general, the results presented in Tables 2 indicates that the pull-in voltages computed using the hybrid scheme are very close to those presented in the literature. Thus, the basic validity of the proposed modeling method is confirmed.

Table 1 Material and geometry parameters of micro circular plate models

Parameters	Model 1	Model 2
	Value	Value
Young's modulus (E) (GPa)	169	130
Poisson's Ratio (ν)	0.3	0.23
Density (ρ) (Kg/m^3)	2.33×10^3	2.33×10^3
Permittivity of free space (ε_0) (F/m)	8.8542×10^{-12}	8.8542×10^{-12}
Thickness of circular plate (h) (μm)	20	3
Initial gap (G) (μm)	1	1
Radius of circular plate (R) (μm)	250	200

Figure 2 shows that for a constant residual stress, the pull-in voltage increases with an increasing gap height. This result is to be expected since the electrostatic force between the upper and lower electrodes reduces as the separation distance between them increases, and hence a higher electrical voltage is required to induce the pull-in event. In addition, it is seen that for a constant initial gap height, the pull-in voltage increases as the residual stress changes from a negative value to a positive value. Note that in this case, the Model 1 parameters apply and the plate is actuated using a DC voltage only.

Table 2 Comparison of present analytical results and literature results for pull-in voltages of Model 1 micro circular plate.

	Analytical results			Deviation	
	Hybrid Method (H.M.) (Model 1)	Osterberg's Model [7]	Reduced-order Model [7]	$\triangle e_1$ (%)	$\triangle e_2$ (%)
Pull-in Voltage (V)	363	362	364	0.27	0.27

$$\Delta e_1\ (\%) = \frac{|\text{Osterberg's Model-H.M.}|}{\text{Osterberg's Model}} \times 100\%$$

$$\Delta e_2\ (\%) = \frac{|\text{Reduced-order Model-H.M.}|}{\text{Reduced-order Model}} \times 100\%$$

Figure 3 shows the variation of the center-point deflection of the Model 2 micro circular plate over time as a function of the AC voltage. (Note that the DC voltage is assigned a constant value of 4.5 V and the residual stress and AC frequency are assumed to be $T_r = 0$ and $\overline{\omega} = 1$, respectively.) The results show that as the AC voltage increases, the center-point deflection of the micro circular plate also increases as a result of the enhanced electrostatic coupling effect. It is observed that the magnitude of the center-point deflection increases nonlinearly with an increasing AC voltage due to the micro circular plate coupling effect. Finally, it is seen that at lower values of the dimensionless time, i.e., $\overline{t}$ <500, the peak values of the center-point deflection are lower than the remaining peaks in the corresponding profile. This is thought to be the result of an instability of the DC voltage once the voltage is initially applied.

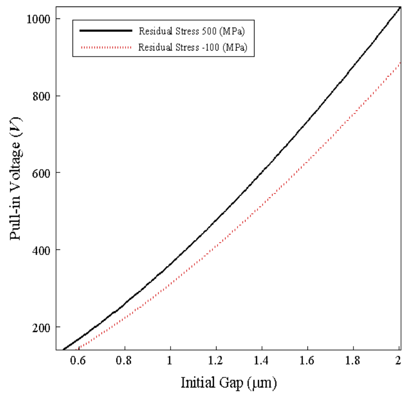

Fig. 2. Variation of pull-in voltage with initial gap height as function of residual stress

Figure 4 shows the variation of the center-point deflection of the Model 2 circular plate over time given AC voltages of 1.61 V, 1.62 V and 1.63 V, respectively. Note that the DC voltage is equal to 4.5 V, the residual stress is equal to $T_r = 0$, and the AC voltage frequency is equal to $\overline{\omega} = 1$. The results clearly show that for an AC voltage lower voltage of 1.62 V, the micro circular plate oscillates in a stable manner about the equilibrium deflection point. However, when the AC voltage is increased to 1.63 V, the pull-in phenomenon occurs, and the micro circular plate makes transient contact with the lower electrode.

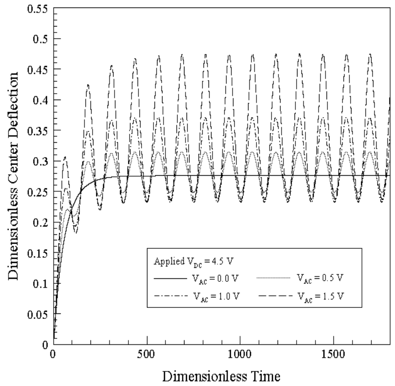

Fig. 3. Variation of dimensionless center-point displacement over time as function of AC voltage. (Note that DC voltage = 4.5 V and $\overline{\omega} = 1$.)

5. Conclusions

This study has analyzed the nonlinear dynamic behavior of an electrostatically-actuated micro circular plate using a hybrid numerical scheme comprising the differential transformation method and the finite difference method. The validity of the proposed scheme has been confirmed by comparing the predicted value of the pull-in voltage for the plate with the results presented in the literature.

In addition, the effects of the residual stress and initial gap height on the pull-in voltage have been systematically explored. The results have shown that for a constant residual stress, the pull-in voltage increases with an increasing gap height. In addition, the pull-in voltage increases as the residual stress changes from a negative value to a positive value. Finally, it has been shown that the use of an AC actuating voltage with an appropriate magnitude and frequency in addition to the DC driving voltage provides an effective means of tuning the dynamic response of the micro circular plate. Overall, the numerical results presented in this study show that the hybrid differential transformation method and finite difference method provides an accurate and computationally-efficient means of analyzing the nonlinear dynamic behavior of the micro circular plate structures used in many of today's MEMS-based actuator systems.

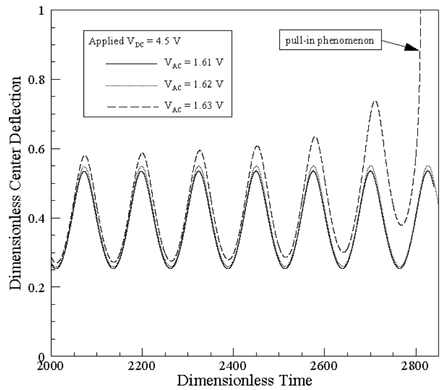

Fig. 4. Variation of dimensionless center-point displacement over time for AC voltages of 1.61 V, 1.62 V and 1.63 V, respectively. (Note that DC voltage = 4.5 V and $\bar{\omega}=1$.)

6. Acknowledgment

Acknowledgments: The authors gratefully acknowledge the financial support provided to this study by the National Science Council of Taiwan under Grant Number NSC 101-2221-E-018 -007 –

7. References

1. V. M. Varadan, K.J. Vinoy and K.A. Jose, RF MEMS and Their Applications, Wiley, New York **,2003**
2. E.S. Hung and S. D. Senturia, Extending the travel range of analog-tuned electrostatic actuators, J. Microelectromeehanieal Systems, 8(4), pp. 497-505 **,1999**
3. D. Gang, W. Kai-Tak, Analysis of One-Dimensional and Two-Dimensional Thin Film “Pull-in” Phenomena Under the Influence of an Electrostatic Potential, ASME J.Appl. Mech., 74, pp. 927-934 **,2007**
4. A. H. Nayfeh, M. I. Younis, and E. M. Abdel-Rahman, Reduced-order models for MEMS applications, Nonlinear Dyn., 41, pp. 211–236 **,2005**
5. M. I. Younis, E. M. Abdel-Rahman and A. H. Nayfeh, Dynamic Simulations of a Novel RF MEMS Switch, NSTI-Nanotech, 2, pp. 287–290,**2004**
6. M. I. Younis, Investigation of the mechanical behavior of microbeam-based MEMS devices, Master of Science Thesis, Virginia Polytechnic Institute and State University **,2001**
7. G. W. V. Vogl and A. Nayfeh, A reduced-order model for electrically actuated clamped circular plates, J. Micromech. Microeng. 15, pp. 684–690 **,2005**
8. C. K. Chen, H. Y. Lai and C. C. Liu, Application of hybrid differential transformation/finite difference method to nonlinear analysis of micro fixed-fixed beam, Microsyst Technol, 15, pp. 813–820 **,2009**
9. C. K. Chen, H. Y. Lai, and C. C. Liu, Nonlinear Micro Circular Plate Analysis Using Hybrid Differential Transformation / Finite Difference Method, CMES: Computer Modeling in Engineering & Sciences, 40 (2), pp. 155-174 **,2009**
10. C. K. Chen and H. Y. Lai and C. C. Liu, Nonlinear Dynamic Behavior Analysis of Micro Electrostatic Actuator based on a Continuous Model Under Electrostatic Loading, ASME Journal of Applied Mechanics, 78, pp. 031003–1~031003–9 **,2011**
11. C. C. Liu, S. C. Yang and C. K Chen, Nonlinear dynamic analysis of Micro Cantilever Beam Under Electrostatic Loading, Journal of Mechanics, 28, pp:559~566 **,2012**
12. C. K. Chen, H. Y. Lai and C. C. Liu, Numerical analysis of entropy generation in mixed convection flow with viscous dissipation effects in vertical channel, International Communications in Heat and Mass Transfer, 38, pp. 285–290,**2011**

微圓板致動器之吸附電壓預測與非線性動態分析

劉晉嘉[1]、吳浩楷[1]
[1]國立彰化師範大學 工業教育與技術學系
國科會計劃編號：NSC 101-2221-E-018 -007

摘要

本研究以混合之微分轉換與有限差分法來分析微圓板靜電致動器所產生的複雜非線性耦合行為，此肇因於微結構與靜電場的耦合效應、非線性靜電力、殘留應力(residual stress)效應等，使得研究分析相當複雜，因此並不適用一般的解析方法，如： Galerkin method 與微擾法 (Perturbation method)。本研究採用混合法，和文獻中各種數值方法進行吸附電壓值比較其誤差不超過0.27%。

同時，此計畫亦針對在直流與交流驅動電壓結合下之微圓板靜電致動器的電極間距與吸附電壓(pull-in voltage)等參數進行動態特性研究，利用交流驅動電壓(AC voltage)之靜電力來驅動微結構系統，其主要優點在於透過交流驅動電壓大小的改變或頻率的變化就可以調整微結構系統振動的幅度。

關鍵字 ：微圓板、吸附電壓、微分轉換法、混合法、有限差分法